

K. Marx's "Mathematical Manuscripts": from the history of creation to understanding the theoretical significance.

(A Review Essay)

Karl Marx: Mathematical Manuscripts. Second revised Edition. Aakar Books 2018. Karl Marx and Mathematics: a Collection of Texts in Three Parts. Aakar Books 2019. Edited by Pradip Baksi.

It is difficult to name any other work by Karl Marx, the publication of which would have been conceived as thoroughly as it was with the mathematical manuscripts of K. Marx, published in India in 1994 and 2018. Interest in the mathematical manuscripts of K. Marx has not abated for more than a century. This part of the handwritten heritage of K. Marx has not yet been published in full, although in comparison with, for example, economic manuscripts, the amount of Marx's mathematical papers is much more modest.

In 1994, an Indian researcher Pradip Baksi edited his English translation of the independent mathematical manuscripts of K. Marx, a description of the entire complex of mathematical works of Marx from 1850s to 1880s, a historical-mathematical appendix, and also supplied his edition with an additional section, which included fragments of the correspondence between Marx and Engels on mathematics, testimonies of F. Engels and P. Lafargue on the topic of “K. Marx and Mathematics”, articles by some Soviet mathematicians and philosophers with an assessment of the mathematical creativity of K. Marx. The translator also included here his own study “On the Problem of situating Marx’s Mathematical Manuscripts in the history of Ideas”[1]. It was published by the Viswakos Parisad of what was then called Calcutta (now Kolkata).

In 2018, he prepared a second edition of the book Karl Marx “Mathematical Manuscripts”, significantly changing its original concept of 1994. He not only included new research on mathematical manuscripts in his edition, but also changed the main message of his work by naming the German mathematician Emil Julius Gumbel (1891–1966) as the first

editor of K. Marx's mathematical manuscripts, placing Sofya Aleksandrovna Yanovskaya (1896–1966) in second place. This review will focus on proving the historical and conceptual invalidity of such a statement.

The work of Pradip Baksi was published in two volumes, with the manuscripts themselves (2018) supplemented by a book “Karl Marx and Mathematics. A collection of Texts in Three Parts” (2019), which generally reproduced the relevant section of the first edition “Marx and Mathematics”, but also included significant additions. Both books were published by the Aakar Books of Delhi.

Pradip Baksi's work is not the first edition of Marx's mathematical legacy in English. In 1983, New Park Publications in London published an edition of Marx's independent mathematical manuscripts with an appendix of several letters from Marx and Engels and articles by S.A. Yanovskaya, E. Kolman and C. Smith.

Pradip Baksi edited both Marx's independent manuscripts on mathematics and a description of the entire complex of Marx's mathematical works from the 1850s to the end of his life.

Of particular interest is the second part with a description of the mathematical manuscripts, which was hardly accessible to the English-speaking reader until 1994. The basis for these editions – the translation into English of K. Marx's mathematical manuscripts – was the bilingual Russian-German edition of K. Marx's book “Mathematical Manuscripts”. Moscow: Nauka, 1968, carried out in the USSR under the supervision of the Institute of Marxism-Leninism, CC CPSU and executed by the eminent mathematicians S.A. Yanovskaya and K.A. Rybnikov.

In the second book “Karl Marx and Mathematics” Pradip Baksi has collected and edited excerpts from the correspondence between K. Marx and F. Engels that explain the subject area, the reasons for Marx's studies in mathematics, and the main milestones of these studies. They are followed by the testimony of F. Engels, as the first biographer of K. Marx, on the reasons and significance of Marx's studies in mathematics, and the memoirs of P. Lafargue, which spoke about the importance that Marx attached to mathematics for the sciences.

He has included in the collected papers “Marx and Mathematics” an extensive bibliography of publications of

mathematical manuscripts in the world, a bibliography of research literature on mathematical manuscripts since the 1930s, as well as the publication of a number of the most famous articles by Soviet and others mathematicians and philosophers on mathematical manuscripts.

Finally, the new edition of 2019 brings together articles on the hundred-year history of publishing mathematical manuscripts, including his own vision of this problem. This part of Pradip Baksi's edition fundamentally distinguishes the second edition from the first, and we will dwell on this part of his work in more detail.

Modern studies of the mathematical manuscripts of K. Marx follow three directions. The first is the study of the history of K. Marx's creation of his works on mathematics. The second is the study of the history of the publication of K. Marx's mathematical manuscripts. Finally, the third is an assessment of K. Marx's contribution to mathematics, consideration of individual problems associated with the manuscripts, in particular, a comparison of G.W. Hegel's and K. Marx's approaches to mathematics, the concept of differential by K.

Marx and the mathematicians of his time, etc. Almost all of these directions, to varying degrees of detail, were covered in the two books of 2018 and 2019.

It is in this sense that Pradip Bakshi's work could have become an indispensable and most complete source of documents on Marx's mathematical manuscripts for specialists and anyone interested in the history of Marxism, in particular, the hitherto little-studied area that is mathematical manuscripts, if, to our greatest regret, there were not for a number of inaccuracies and errors against historical facts that reduce the significance of the work. To explain this therefore, we will dwell on each of the three areas to see how Pradip Bakshi has covered them in his publication.

The history of the writing of manuscripts on mathematics by K. Marx is presented in the book, as already mentioned above, by a selection of documentary materials – fragments from the correspondence of K. Marx, F. Engels and third persons, as well as excerpts from Engels's “Anti-Dühring”, his speech at Marx's funeral and the memoirs of P. Lafargue. As for the correspondence, Pradip Bakshi cites excerpts from letters

from K. Marx to F. Engels dated January 11, 1858, July 6, 1863, May 31, 1873, November 22, 1882, from F. Engels to F. Lange dated March 29, 1965, and from F. Engels to K. Marx dated August 18, 1881 and November 11, 1882. In this way, he expands the range of letters from K. Marx and F. Engels on mathematics traditionally cited by publishers.

However, it should be recalled that these letters do not exhaust the coverage of K. Marx's and F. Engels's correspondence in mathematics. K. Marx's studies in mathematics are also evidenced, in particular, by K. Marx's letters to F. Engels dated November 23 1860, May 20, 1865, August 25, 1879, and a remarkable appendix to the letter on the tangent, the more precise dating of which places the letter in all likelihood in 1863[2]. Also of interest on this topic are F. Engels's testimonies in his letters to K. Marx dated May 30, 1864, to F. A. Sorge dated June 29, 1883, and Laura Lafargue dated June 24, 1883.

editor Pradip Baksi does not provide any more research on the history of Marx's writing of his mathematical manuscripts. Information about this can nevertheless be obtained from the

preface by S.A. Yanovskaya[3] and independently researched based on the study of the description of mathematical manuscripts given in the first volume.

The second important direction of the study of Marx's mathematical manuscripts is **the reconstruction of the history of their publication**. Two works in the publication are devoted to this problem: Introduction, 2018, (author – Pradip Baksi)[4] and the article Vogt A. “Emil Julius Gumbel (1891–1966): The first editor of the mathematical manuscripts of Karl Marx”[5].

In his reasoning, Pradip Baksi clearly agrees with the conclusions made in Vogt's article. At least, he does not doubt Vogt's conclusions and does not provide new data on the first stage of the preparation of mathematical manuscripts for publication by the German mathematician Emil Julius Gumbel. Moreover, on the title page of the main volume of the Mathematical Manuscripts edited by Pradip, the names of two *antagonists* in the understanding of mathematical manuscripts appear: Emil Julius Gumbel and Sofya Aleksandrovna Yanovskaya. After all, it is A. Vogt who insists on the priority of E.J. Gumbel in the publication of manuscripts.

We have already said earlier that the edition of mathematical manuscripts of K. Marx, carried out by Pradip Baksi, is a translation of the book K. Marx. “Mathematical Manuscripts”. Moscow: Nauka. 1968. In both the 1968 edition and here the structure and text of the manuscripts coincide, and the translation corresponds to the text. Pradip Baksi also translates the preface of S.A. Yanovskaya to the 1968 edition. In the introductory notes to the first edition of the book in 1993, Pradip Baksi wrote: “In K. Marks, *Matematicheskie Rukopisi* (“Nauka” M. 1968), Marx’s own texts have been published both in the original language (mainly German, but in places French, English or a mixture of two or more of these languages) and in Russian translation. However, the preface, editorial comments, notes and appendices are all in Russian only. Hence, Russian is the only single language through which the entirety of this volume becomes accessible. The present translation has throughout followed the texts, comments, notes and appendices in Russian[6].

The question arises: if the title of the second edition mentions the name of Emil Julius Gumbel as the first publisher

of Marx's mathematical manuscripts, then one would like to know what exactly of his work as an editor was taken into account in this book. But apart from the publication of E.J. Gumbel's article "On the Mathematical Manuscripts of K. Marx (Communication)" (Letopisi Marksizma [Chronicles of Marxism]. (Notes of the Institute of K. Marx and F. Engels). Moscow-Leningrad: State Publishing House, 1927. V. 3), 56–60 and A. Vogt's article on the preliminary stage of work on systematizing the manuscripts and preliminary plans for their publication, nothing is known about this work in principle for the sole reason that the work did not go beyond preliminary plans.

Let us briefly describe E.J. Gumbel's work with K. Marx's mathematical manuscripts. In October 1925, Privat-docent E.J. Gumbel arrived in Moscow, using the opportunities of his six-month sabbatical at Heidelberg University to systematize Marx's mathematical manuscripts and prepare a certain part of them for publication. It was presumably Monday, October 12, 1925. E.J. Gumbel's stay in Moscow, as he had planned from the very beginning, was not long. Already at the beginning of April

1926, i.e. six months after his arrival, E.J. Gumbel completed his work in Moscow and returned to Germany. There is documentary evidence of his arrival and departure. Therefore, the dates 1925–1927, repeatedly indicated in Pradip Baksi's book, as the period of E.J. Gumbel's work on the publication of K. Marx's mathematical manuscripts at the Institute of K. Marx and F. Engels in Moscow (IME) does not stand up to criticism, as does the reference to E.J. Gumbel's alleged work in Moscow on the publication of manuscripts in 1930[7].

E.J. Gumbel repeatedly wrote to IME asking to come and continue working on mathematical manuscripts. Thus he was ready to come to the USSR to prepare volume XVI of Complete Works of K. Marx and F. Engels in the original languages (Marx-Engels-Gesamtausgabe/MEGA¹) from the end of February to the beginning of May 1927, to which he received the answer that the preparation of this volume had been postponed for a long time.

In 1929, E.J. Gumbel wrote to the IME that he would like to use his vacation from the beginning of August to the beginning of November of the current year and asked to convey

his request to the director of the Institute D.B. Ryazanov about whether he could use this time to work at the Institute. The letter contains the resolution “sehr höfliche ablehnende Antwort” – politely refused.

E.J. Gumbel came to the USSR for the second time, but his stay in Moscow was limited to three weeks, which he himself wrote about in the magazine “Die Weltbühne” in the article “Moscow 1932”[8]. Therefore, there is no reason to believe that E.J. Gumbel used his second stay in the USSR to work on mathematical manuscripts. In addition, anyone who is somewhat familiar with the publication of manuscript heritage of K. Marx will understand that six months is an unrealistic period for preparing manuscripts for publication, from their deciphering, systematization, to historical and critical commentary. Moreover, during this period, the German mathematician and publicist wrote another 15 journalistic articles and published two scientific works, and this also takes time[9].

As for the results of the first task of E.J. Gumbel – systematization of mathematical manuscripts of K. Marx and selection of a number of drafts for publication, this can be

judged on the basis of the communication of E.J. Gumbel “On the mathematical manuscripts of K. Marx”^[10], published by the “Letopisi Marksizma” (Chronicles of Marxism) in 1927, and the correspondence of the German mathematician with the staff of the IME, stored in the Russian State Archive of Social and Political History (RGASPI). He divided all the material into four groups: A) calculations, B) excerpts, C) sketches and D) independent works. In sections A) and B) – calculations and excerpts – he divided the manuscripts by branches of mathematics: I) arithmetic, II) geometry, III) algebra, IV) differential calculus; separately highlighting V) Taylor's theorem.

As E.J. Gumbel himself asserted, the principle of systematization for him was an exclusively formal approach. He believed that only those manuscripts that do not contain calculations or excerpts can be considered as independent works of K. Marx. He did not consider the publication of other manuscripts different from these “independent” notes to be expedient.

From the communication of E.J. Gumbel in the “Letopisi Marksizma” (Chronicles of Marxism) it becomes clear what independent notes and where they were intended to be published. He wrote about one note by K. Marx on the differentiation of the product, which was planned to be published in the “Archive of K. Marx and F. Engels” and 13 manuscripts by K. Marx on mathematics of more than 200 typewritten pages – in Volume XVI of the Complete Works of K. Marx and F. Engels in the original languages (*MEGA*^I): “His [Marx’s] independent work on the differentiation of products is of great interest; it will be published in German, in the next issue of ‘Arkhir K. Marksa I F. Engelsa’. All the works indicated through the numbers 1–13 above are ready for the press, and these will appear in Volume XVI of the Collected Works”[11].

E.J. Gumbel wrote that the papers were ready for publication, but, considering the short period of their actual preparation by 1927, it can be assumed that the author had in mind mainly the readiness of their deciphering. For publication in Volume XVI of *MEGA*^I, E.J. Gumbel selected the following, as he considered, independent works by Marx:

1) “Two notes within the excerpt ‘Algebra I’. From the chapter ‘First elementary emergence of $\frac{a}{0} = \infty$ and $\frac{0}{0}$ in ordinary algebra’ and divergent series”[12].

2) The nature of differentiation explained through differentiation of power.

3) Differentiation of the simplest functions.

4) Differentiation of the product. Two additions by Engels.

5) A variant of the preceding text.

6) Additionally on the differentiation of the product[13].

7) “Taylor's Theorem”[14].

8) Derivation of Taylor's Theorem[15].

9) Taylor's Theorem in its various forms.

10) “Derivation of Taylor's Theorem by Lagrange based upon algebraic foundation”[16].

11) MacLaurin's theorem.

12) “Taylor's Theorem, MacLaurin's Theorem. Lagrange's Theory on derived functions”[17].

13) “From Newton's communication of 1669 ‘De analysi aequationes numero terminorum infinitas’ addressed to the President of the Royal Society of London”[18].

In order to make the choice of the German mathematician clear, we have provided a number of references to pages of the Delhi edition, which, in our opinion, E.J. Gumbel undoubtedly had in mind. As can be seen, many of them were later recognized not as independent works, but as Marx's excerpts from mathematical literature.

However, soon, as was already known in October 1926, the IME postponed the plan to publish mathematical manuscripts in Volume XVI of the *MEGA*¹: “There has been no mention of the 16th volume for quite some time”.

The plans to publish one or two of Marx's sketches on mathematics and an extensive commentary on them and on Marx's other manuscripts on mathematics by E.J. Gumbel in the

German and Russian editions of the “Archives of K. Marx and F. Engels” were still in force at this stage.

As follows from the already mentioned report in “Letopisi Marksizma” and from the correspondence of E.J. Gumbel, and this is very important to emphasize and further to keep in mind, it was not a question of publishing 13 manuscripts, but only one or two final drafts of K. Marx: one of which was undoubtedly the manuscript that E.J. Gumbel called “Differentiation of the Product”. It subsequently received the editorial title “On the Differential” (the manuscript had no title, but bore the name of the addressee “For Fred”, i.e. for F. Engels). In it, Marx, using the theorem on the derivative of a product, expounded his own method of differentiation. The second planned manuscript could be a draft of an addition to this manuscript.

This is evidenced by the descriptions of the manuscripts made by E.J. Gumbel. He wrote that the manuscript consists of two parts, is written cleanly, the first part has pagination 1–13, the second – 1–5, page 4 is missing. Substantively, this is:

I. pages 1–6: differentiation of the product,

II. pages 6–13: the nature of the differential coefficient,

A. pages 1–3 differentiation of a quotient (fraction)

B. pages 3–5 Differentiation of an implicit function.

This description (I–II) corresponds to the manuscript “On the Differential”, which has two parts, 13 pages[19] and has at the end an indication of the intention to write a continuation in Part III. The second part (A–B) coincides in its headings with the “Additions to the manuscript on the differential”[20].

It was this article that E.J. Gumbel prepared for the “Archive of K. Marx and F. Engels”, it was this work that was prepared for publication, but never saw the light of day. This article E.J. Gumbel prepared for the “Archive of K. Marx and F. Engels” and entitled “Mathematical Manuscripts of Marx and Engels. Published by E.J. Gumbel. Heidelberg” (Die mathematischen Manuskripte von Marx und Engels. Herausgegeben von E.J. Gumbel. Heidelberg).

Only the first page with the aforementioned title and the beginning of the introduction has survived from this manuscript, in addition to the above-mentioned message from the “Letopisi

Marksizma”, which, according to E.J. Gumbel, was also supposed to be included in the article. Directly under the title was a list of the sections of the article, revealing its actual content:

1. List of manuscripts.
2. Description of drafts.
3. Reproduction of two drafts.

Let us repeat that it was precisely the article by the German mathematician that was in question, and that the two drafts written by K. Marx made up the smaller part of it. The title of Gumbel's article did not satisfy the leadership of the IME. True, it is not clear from the correspondence what was proposed as a replacement; it is possible that it was the title “Marx's Studies in Mathematics” (Mathematische Studien...). Discussing this issue, Gumbel wrote that he would strongly recommend keeping the original title; because the Volume XVI – optimistically speaking – will not be published in the foreseeable future, so for now this article will remain the final and conclusive of what is available from this entire complex, so he would not like to reduce the

importance of the article by naming its actual content in the subtitle. E.J. Gumbel made numerous corrections to the layout of his article, added new pages, notes, rearranged paragraphs, etc., which delayed publication. The article was transferred from one issue of the “Archive” to another, but as a result the article never saw the light of day.

E.J. Gumbel was not the only foreign specialist at IME. In particular, more than ten emigrants from Hungary worked at IME at that time, who were attracted here by the deputy director of IME E. Czóbel (1886–1953)^[21].

On p. xiv of Pradip Bakshi's book “Karl Marx. Mathematical Manuscripts”, the collaboration of E.J. Gumbel with E. Czóbel., R. Matejka and R.S. Bogdány is mentioned. To be fair, R. Matejka and R.S. Bogdány is one and the same person. An employee of the IME archive (1922-1930) Rózsa Bogdány (Matejka) (1903–1951), took part in deciphering the manuscripts from 1925, apparently not even having a mathematical education by that time. Rózsa Bogdány was the first wife of the Hungarian writer János Matejka (1895–1940) and for some time bore his surname. They were among the 400

people who arrived in the USSR in 1921 as a result of the agreement between the RSFSR, the Ukrainian SSR and Hungary on the exchange of prisoners of war and interned civilians on July 28, 1921^[22]. Both worked at the IME for some time. Studying the mathematical manuscripts of K. Marx could not help but have an impact on the fact that only after they had been ready to publish did R.S. Bogdány enter and graduate from the Mechanics and Mathematics Department of the Moscow State University (Mechanics and Mathematics Department of Moscow State University) in 1936.

It follows that the German mathematician E.J. Gumbel can rightfully be considered one of the first scientists who began working on the systematization of the mathematical manuscripts of K. Marx and the preparing of a certain part of them for publication, while calling him their first editor is fundamentally incorrect. This work was not the work of one person. The publication of Marx's mathematical manuscripts required an understanding of their internal logic and structure and was the result of the work of many specialists: textual decipherers and mathematicians who gave the manuscripts both a professional

historical-mathematical and philosophical assessment. It lasted with a break for the years of the Great Patriotic War in the USSR from the 1930s to the 1960s.

In his 2019 preface, Pradip Baksi attributes the suspension of work on the publication of the manuscripts after 1926 to the arrest of D. B. Ryazanov. It seems that this is not the case. It was D.B. Ryazanov himself who was not satisfied with the results of E.J. Gumbel's work. Neither the manuscripts themselves selected for publication, nor E.J. Gumbel's detailed article were published, as they seemed unconvincing to D.B. Ryazanov did not prove the scientific significance of these works of K. Marx, which made the publication of disparate sketches devoid of political and ideological meaning.

The then head of the research library of K. Marx and F. Engels; at the IME (let us emphasize, the head, not the publisher), the Soviet mathematician and philosopher E. Kolman (1892–1979) believed that D.B. Ryazanov delayed publishing the manuscripts because he was not sure of their scientific significance and was afraid of harming K. Marx's authority. Later, E.J. Gumbel was denied further work at the IME. One can

only assume that the reason for this was precisely the formal approach of the German mathematician to processing the manuscripts, which could not but subsequently affect the corresponding assessment of the significance of Marx's manuscripts by the scientific community.

The arrest of D.B. Ryazanov did not delay, but rather accelerated the publication of manuscripts, since the new leadership attracted a new team of specialists for this work. On November 3, 1931, the IME and the V.I. Lenin Institute were merged (IMEL), and at the same time the nature of the publication “Archive of K. Marx and F. Engels” was changed, and the article by the German mathematician, which he was preparing for publication in this publication, no longer fit into its format: “From now on, only manuscripts from the literary heritage of the founders of Marxism were published in it, and research articles, reviews, etc. were no longer published. At the same time, the publication of the journal “Letopisi Marksizma” ceased^[23].

The publication of a number of Marx's mathematical manuscripts in Russian was prepared by Soviet mathematicians

under the leadership of S.A. Yanovskaya by 1933. They were first published in 1933 in the journal “Pod znamenem marksizma. Filosofskiy i obshestvenno-politicheskiy zhurnal (№ 1. 1933) [Under the banner of Marxism. Philosophical and socio-political journal. (No. 1. 1933), 14] and in the collected papers “Marxism and Natural Science”. It was then that the manuscripts were further deciphered, the main areas of Marx's studies in this discipline were determined, and thus the logic and structure of Marx's mathematical heritage, and in addition, some manuscripts were translated into Russian.

If we compare this publication with the subsequent one from 1968, it should be noted that already in 1933 all of Marx's works that were later included in the first part of the 1968 book were published. In 1968 they were only re-edited and systematized. The 1933 edition not only contained the manuscripts themselves, but the notes also provided comparisons of individual passages with their draft versions. The difference in the systematization of Marx's manuscripts given by S.A. Yanovskaya from that proposed six years earlier by E.J. Gumbel is interesting.

The introductory article to the manuscripts states: “Marx’s mathematical works, which are independent in nature, are divided into two parts. The manuscripts dating back to an earlier period deal with lines by Taylor and MacLaurin and Lagrange’s “Theory of Analytic Functions”. In later manuscripts, Marx gives his own justification for differential calculus. It is precisely this that is discussed in the correspondence between Marx and Engels (letters of August 18, 1881 and November 21 and 22, 1882). This last part of the work is in turn divided into three parts, of which the third (historical essay) is only in rough draft form”[24].

Thus, from the very beginning, S.A. Yanovskaya gave not a formal, but a substantive classification of manuscripts, and declared the presence of Marx's own contribution to mathematics through his own substantiation of differential calculus. This classification singled out from the entire array of Marx's mathematical manuscripts the essential problems that lay at the center of his attention, and also found its confirmation in Marx's characteristic approach to the research method and the method of presentation. And to this day, in RGASPI, the

detailed division of manuscripts into separate fragments is subordinated to this very understood structure of manuscripts, which is not the case in Amsterdam International Institute of Social History (IISG) where are the original mathematical manuscripts of K. Marx kept; editorial headings were compiled for the manuscripts when they were published, which makes it easier to work with them. The following manuscripts were included in the 1933 publication:

I. Derivative and symbolic differential coefficient.
Algebraic differentiation of the simplest functions[25].

About replacing the symbol $\frac{0}{0}$ with the symbol $\frac{dy}{dx}$ (Notes)

Comparison of Marx's method of differentiation with
D'Alembert's method

II. Differential and differential calculus [26]

Revolution in Method. Differential

Additional Notes on Differentiation of a Product [27]

Variant of work on differential

First draft of work on differential [28] .

III. Historical essay[29].

II. Historical course of development.

A negative assessment of E.J. Gumbel's work was voiced in S. A. Yanovskaya's article on mathematical manuscripts in 1933. The attentive reader understands that in criticizing the article in the "Letopisi Marksizma", S.A. Yanovskaya has in mind precisely its author, E.J. Gumbel, and not the leadership of the IME. Criticism of E.J. Gumbel was also heard in other publications. Therefore, it would be completely inappropriate to reproduce it in the 1968 edition (essentially in the second Russian edition of the manuscripts), when the initial stage of work on the systematization of mathematical manuscripts was overcome and a fundamentally new edition was prepared. The accusations against Soviet mathematicians that they allegedly did not mention in the 1968 edition are absolutely unfounded.

S.A. Yanovskaya was not the kind of person who ignores the work of her colleagues. Given her high assessment of the work of the decipherers of K. Marx's manuscripts, it is worth

mentioning the decipherers, and among them not only Róza Bogdány, who worked with E.J. Gumbel, but also Yakov Wildhaber. Yakov Wildhaber (Jakob Wildhaber)[30] born in 1887 in Switzerland (Sargans), he was a mathematics teacher by profession. In 1924, he came to the USSR as part of a group of Swiss skilled workers and engineers, which was headed by the figure of the international socialist and communist movement and friend of V.I. Lenin Fritz (Friedrich) Platten (1883–1942). The purpose of coming to the USSR was to work in the Swiss cooperative “Solidarity” organized on the Volga. Soon after the move, J. Wildhaber got a job in Saratov as a mathematics teacher. Then he moved to Moscow and in 1931–1935 worked at the IME, deciphering Marx's mathematical manuscripts, after which he transferred to work at Moscow State University. In 1938, J. Wildhaber went missing^[31].

It was with the publication of 1933 that the study of these documents, their commentary from a philosophical and mathematical point of view began. One of the first researchers was a Dutch-born American mathematician, historian of mathematics Dirk Jan Struik (1894–2000), who in 1948, based

on the publications of 1933, introduced the world to these manuscripts of K. Marx.

Pradip Bakshi's reasoning regarding the period 1933–1968 in the work on the publication of K. Marx's mathematical manuscripts also deserves a separate comment. The Russian State Archive of Social and Political History (RGASPI) contains evidence of this work[32]. In particular, it has been documented that work on preparing the mathematical manuscripts of K. Marx for publication was started at IMEL in October 1950 and was carried out directly in the IMEL archive.

On February 4, 1953, S.A. Yanovskaya and K.A. Rybnikov sent a report to the IMEL Directorate entitled “On Marx’s Mathematical Manuscripts”^[33], in which they dwelled in detail on the structure and significance of Marx's mathematical manuscripts, contrasting their point of view with E.J. Gumbel's approach to the systematization and dating of Marx's mathematical works. “It is not surprising”, wrote the authors of the report on E.J. Gumbel's study of Marx's mathematical manuscripts, “that all this 'work' ended with a libel on Marx's manuscripts, published in the *Letopisi Marksizma*, III, 1927, and

containing an assessment of these manuscripts as Marx's student exercises”^[34]. From then on, work on the manuscripts was carried out at IMEL until 1968. After such an assessment, it is obvious that the editors S.A. Yanovskaya and K.A. Rybnikov would have been shocked if their names had been mentioned together with the name of E.J. Gumbel, as the alleged first editor of Marx’s mathematical legacy.

The 1968 edition consisted of two parts: the first part practically reproduced the 1933 edition of the manuscript “On the Concept of a Derivative Function”, “On the Differential”, “From the History of Differential Calculus”. However, to the already known manuscripts, a description of those manuscripts that were written by Marx in other periods of time was added. Thus, Pradip Baksi's edition is a reproduction of the book prepared by S.A. Yanovskaya and K.A. Rybnikov, it is based on their conclusions and has nothing to do with the mathematician E.J. Gumbel, indicated in the title of his book as their first editor. The merits of this scientist are already great, so attributing to him the publication of K. Marx's mathematical manuscripts, in our opinion, is superfluous.

After the 1968s, their mass translation into different languages began. A characteristic feature of these editions was that they were all limited to the first part of the manuscripts, that is, they did not take into account the entire history of their creation. This part of the book was omitted by the publishers, and we repeat that Pradip Baksi's merit is that he was the first to publish something that had not come to the attention of researchers until now. A detailed review (description) of Marx's various manuscripts on mathematics in the second part of the book gives the researcher (not just the reader, because the publication does not promise easy reading) the opportunity to become acquainted with the history of Marx's studies in mathematics beginning in the 1850s.

Returning to the period of 1933–1940s, we must not forget that in addition to the internal policy of the USSR and the period of repressions, which undoubtedly could not but affect the work of all, especially ideological institutions, such as IMEL, the USSR survived and victoriously ended the Great Patriotic War of 1941–1945, and many employees of the Institute fell on the war fronts, among them is the excellent decipherer and editor of

the manuscripts of K. Marx and F. Engels – Pavel Lazarevich Weller (1903-1941).

The rise of the fascists to power in Germany dates back to 1933, at the same time the SPD was banned, which undoubtedly influenced the cessation of the publication of *MEGA*¹, which from the very beginning implied international cooperation. The fight against fascism, from which the German mathematician E.J. Gumbel also suffered, forced to emigrate to the USA, unfortunately, does not fit into the reasoning of modern authors trying to explain the delays in the publication of the works of K. Marx and F. Engels, in particular, the cessation of the publication of *MEGA*¹, exclusively by Stalin's repressions.

Pradip Baksi also has some unclear information about the work on mathematical manuscripts in *MEGA*². In the mid-1980s, collaboration on this issue was offered to a group of mathematicians from Moscow State University under the leadership of K.A. Rybnikov, who, as was said earlier, worked together with S.A. Yanovskaya on the publication of mathematical manuscripts of 1968[35]. However, this work did not take place, since it was not possible to create a team of

mathematicians and editors, whose collective efforts could have been used to prepare K. Marx's manuscripts for *MEGA*² taking into account the editorial principles of *MEGA*², which included the reproduction of all variants present in the manuscripts and corrections made during the scientific editing of K. Marx's texts, and then to comment on them in detailed notes.

Based on unverified data, Pradip Baksi presents his vision of the situation that arose at the IML under the Central Committee of the CPSU in the 1990s. It must be acknowledged that these years were not easy for the employees of *MEGA*² as for all citizens of the USSR. However, the present reviewer I.K. Antonova did not leave the IML “in order to survive”, as the author writes. Not being a professional mathematician, let alone a historian of mathematics, she did not consider herself entitled to undertake professional commentary on the mathematical manuscripts of K. Marx. Nevertheless, I.K. Antonova completed the textual work on the manuscripts selected for publication in volume I/28, and left the IML only after she had handed over the text she had typed on the computer, the indexes of variants and corrections, some notes, a draft of the history of the creation of

the text, and the preface for further work to the French researcher Alain Alcouffe, who in 1985 prepared for publication the edition of “Les manuscrits mathématiques de Marx”. Etude st près. Par A. Alcouffe. Premier trad. française. Paris. But work on the publication of the mathematical manuscripts stopped there.

When a new attempt was made to publish the manuscripts within the framework of *MEGA*², which was started by Carl Erich Vollgraf, the publishers doubted the legitimacy of the selection of certain mathematical works by K. Marx for publication in volume I/28. In other words, the question arose again, the answer to which researchers had been looking for almost a century: for what purpose did Marx undertake his research in mathematics and what in the mathematical manuscripts should be considered extracts, and what is his independent contribution to this discipline. At this point, the work on this volume of *MEGA*² stopped. And this question remains open to this day. It is not right to call all those involved in the work on the publication of Marx's mathematical manuscripts members of the preparatory team; they never

worked together; each of them studied this question independently at different periods of time.

The **bibliographical research** of Pradip Baksi, published in the book together with articles on K. Marx's contribution to mathematics, is very valuable. Pradip Baksi provides a bibliography of publications of mathematical manuscripts in different languages, a bibliography of research papers on mathematical manuscripts. The only thing that is puzzling is that the bibliography is published in different parts of the book, and not combined in one section. Then follow the articles themselves by a number of Soviet researchers. Among them are the article by V.I. Glivenko “The Concept of Differential according to Marx and Hadamard”, the article by V.N. Molodshii “Marx's ‘Mathematical Manuscripts’ and Development of History of Mathematics in the USSR”, V.I. Przhemitski, “On the Logical Apparatus working in Marx’s ‘Capital’ and ‘Mathematical Manuscripts’ etc. In addition, Pradip Baksi included in the collected papers his already mentioned work “On the Problem of situating Marx’s Mathematical Manuscripts in the History of Ideas” and Alain Alcouff’s article “The differential calculus,

mathematicians and economists in the nineteenth century: K. Marx and H. Laurent, readers of J.L. Boucharlat”. However, it would be interesting to see among these works the article by Dirk J. Struik “*Marx and Mathematics*” (Science & Society. Vol. 12. No. 1. Winter. 1948. A Centenary of Marxism), 181–196, which for the first time introduced Marx’s mathematical manuscripts to the English-speaking reader.

Of interest are the researches of Pradip Baksi on the mathematical manuscripts of G.W. Hegel, which were mentioned by F. Engels in a letter to F.A. Lange on March 29, 1965: “Hegel knew so much mathematics that none of his disciples was capable of editing the numerous mathematical manuscripts he left behind. The only man who, to my knowledge, has enough understanding of mathematics and philosophy to be able to do so is Marx”[36]. In his collection “Marx and Mathematics”, Pradip Baksi provides an interesting bibliography of works on the topic of “Hegel and Mathematics”[37].

In this review we have not touched upon Pradip Baksi's translation of mathematical manuscripts into English. This

painstaking work will require considerable time from specialists. However, we would like to mention one concept. This is the concept of “inversion of method” (reversal of method). It is of fundamental importance for the entire system of Marx's reasoning about the connection between algebra and differential calculus. With its help, K. Marx not only noted the transition of the algebraic method to the differential method, but also pointed out their interaction within the framework of differential calculus. It plays a significant role in the methodology of K. Marx's scientific knowledge as a whole. The very internal structure of the ascent from the abstract to the concrete is subject to the law of reversal of method.

The notion of inversion of method, to which Marx, and after him Engels, attached fundamental importance in the whole complex of Marx's mathematical research, appeared not only in the rough sketches for the work “On the Differential”, but also when commenting on the excerpted sources. As early as in the excerpts of Sauri and Boucharlat's textbooks, Marx raised the question of a methodological technique that would allow one to move from algebraic to differential methods. The relation

between differential and algebraic methods was formulated in a general form, which was the precondition for the formation of a generalized methodological notion, such as the notion of inversion of method.

In German, Marx uses the phrase “Umschlag in der Methode” and the corresponding verb “umschlagen” for it. However, there is no single term for this concept in English editions of mathematical manuscripts. Pradip Baksi did not escape this either. There are different modes of translation of the concepts “Umschlag in der Methode” into English: inversion of method (P. 19, note, 24 etc.), method reversal, reversal of the method (P. 19). The reasons for this are unclear.

Once K. Marx cited as his motto “De omnibus dubitandum” – “Everything should be doubted”, or “Doubt everything”. This critical attitude was not a simple slogan, but the style of K. Marx’s research, who, getting to the bottom of phenomena, turned to many sources, not trusting one single source, not building his hypotheses on unverified data, it is not for nothing that his works on philosophy and political economy

often begin with the word “Criticism...” However, the editor of mathematical manuscripts Pradip Baksi, unfortunately, more than once in his work trusted conclusions and evidence that were not supported by facts, which led to a distortion of the history of the publication of mathematical manuscripts in his own work. Perhaps, of course, K. Marx himself would have forgiven him, because it was not for nothing that he then wrote about a shortcoming that he was ready to excuse – gullibility. The purpose of this article was to give Pradip Baksi an incentive to perfect his work, which is so remarkable in its conception.

[1] Marx K. *Mathematical Manuscripts*. (Calcutta. 1994), 435 – 447.

[2] Marx K., Engels F., *Collected Works*, (LW, 2010, Vol. 42), 208 – 210.

[3] Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), xv – xxix.

[4] Karl Marx and Mathematics. A Collection of Texts in Three Parts. Ed. by Pradip Baksi (Delhi: Aakar Books 2018), v – xxvii.

[5] Karl Marx and Mathematics. A Collection of Texts in Three Parts. Ed. by Pradip Baksi (Delhi: Aakar Books 2018), 22 – 33.

[6] Marx K. *Mathematical Manuscripts*. (Calcutta. 1994), VIII.

[7] Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), III, XIV.

[8] E.J. Gumbel. *Moskau 1932*. (Die Weltbühne 28-1. 1932), 400.

[9] In the preface, Pradip Baksi writes “The mathematical manuscripts of Karl Marx were first edited during the years 1925–1927 and 1930 by the famous German-American mathematician , statistician, pacifist and political writer Emil Julius Gumbel (1891–1966). The next editor of these manuscripts was the Soviet Russian logician, philosopher and historian of mathematics Sofia Aleksandrovna Yanovskaya (1896–1966)” (Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018, iii).

[10] Gumbel E.J., *O matematicheskikh rukopisiyakh K. Marksa (soobshenie) (Letopisi Marxizma. Zapiski Instituta K. Marksa i F. Engelsa. Kn.3. Moskva-Leningrad 1927)*, 56. (Gumbel E.J., *On the Mathematical Manuscripts of K. Marx (report) (Chronicles of Marxism. (Notes of the Institute of K. Marx and F. Engels). Book 3. M.–L. 1927)*, 56–60.). The author's copy of the journal was sent to E.J. Gumbel on May 25, 1927.

[11] Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), xiv.

[12] Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), 165 sqq.

[13] Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), 51sqq.

[14] Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), 80sqq.

[15] Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), 267 sqq.

[16] Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), 124.

[17] Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), 236.

[18] Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), 117.

[19] Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), 13–27.

[20] See Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), 258 – 265.

[21] Zhelitzky B.Y., Zhelitzky C.B., *Vengerskiye emigracionnye volny i emigranti (seredina XIX – konez 50 godov XX veka)*. (M. 2012), 265. [Zhelitzky B.Y., Zhelitzky C.B. *Hungarian Emigration Waves and Emigrants (mid-19th – late 1950s)*].

(Moscow. 2012), 265.]. Among them, by the way, was the philosopher Georg (György) Lukács. (1885–1971), who worked at IME for a short time.

[22] Dokumenty vneshney politiki SSSR. (T. IV. 19 marta 1921–31 dekabrya 1921 M. 1960), 242–246 [Documents of the Foreign Policy of the USSR. (Vol. IV. March 19, 1921 – December 31, 1921. Moscow. 1960), 242–246]; Sobraniye uzakoneniye i rasporyazheniy Pravitelstva za 1922 g. (M. 1950), 185–188.]; [Collection of Laws and Orders of the Government for 1922. (Moscow, 1950), 185–188].

[23] Literaturnoye nasledstvo K. Marksa i F. Engel'sa. Istoriya publikatsii i izucheniya v SSSR / Sost. Ye. Kandel'. M.: Politizdat, 1969. S. 160. [The Literary Heritage of K. Marx and F. Engels. History of Publication and Study in the USSR / Comp. E. Kandel. Moscow: Politizdat, 1969. P. 160].

[24] Pod znamenem marxizma. Filosofskiy i obshchestvenno-politicheskiy zhurnal. (№ 1. 1933), 14. [Under the banner of Marxism. Philosophical and socio-political journal. (No. 1. 1933), 14].

[25] We will show, using a number of examples, how the manuscripts were systematized in 1933, comparing them with the arrangement of the manuscripts in the 1968 edition. Marx

K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), 5 – 11.

[26] Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), 13 – 27.

[27] Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), 51 – 52.

[28] Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), 28 – 40.

[29] Marx K., *Mathematical Manuscripts*. (Delhi: Aakar Books, 2018), 53 – 70.

[30] Fayet J.-F., Huber P. *Die Russlandschweizer ohne Schutz? Die IKRK-Mission in Moskau als «verdecktes Konsulat» 1921-1938* (Studien und Quellen – Études et sources – Studi e fonti 28, 2002), 153–187. (IKRK – International Committee of the Red Cross).

[31] “Mathematics teacher Jakob Wildhaber arrived in Novaya Lava [a village in the Ulyanovsk region] in 1924, but like other university-educated émigrés, he used his stay in the countryside to look for another job in the city. After only six months, he left Novaya Lava to teach mathematics at the technical institute in Marksstadt [now Marks] on the Volga. From 1931 to 1935, Wildhaber worked as a research fellow at the Marx-Engels Institute in Moscow. He deciphered Marx's manuscripts. The institute's staff, which was responsible for publishing the works

of Marx and Engels, had recently undergone a ‘purge’ of all old employees in the wake of the Menshevik trial, giving Wildhaber an opportunity to advance in his career. At the same time, he was engaged in an activity that later may have cost him his life: ‘Development of my own discovery in the field of artillery calculations. Work completed and handed over to the USSR Defense Commissariat for practical use’. In the spring of 1936, for unknown reasons, Wildhaber left (he was now a Soviet citizen) for Engels, where he began teaching at a secondary school. His mother received her last letter in January 1938. She raised the alarm by writing to the Red Cross. Two letters sent to Engels were returned marked ‘return, left’. Wildhaber is considered missing in action”. (Fayet J.-F., Huber P. *Die Russlandschweizer ohne Schutz? Die IKRK-Mission in Moskau als «verdecktes Konsulat» 1921-1938* (Studien und Quellen – Études et sources – Studi e fonti 28, 2002), 153–187, 164–165.

[32] Also worth noting is the article Laskovaya T.A., Rybnikov K.K., Chernobrovina O.K. *Matematicheskie raboty K. Marksa: stoletnyaja istoriya poiska, rasshifrovki i analiza*. (Voprosy istorii estestvoznaniya i tehniki. T. 42. № 1. 2021), 71–88. [Laskovaya T.A., Rybnikov K.K., Chernobrovina O.K. *Mathematical works of K. Marx: a hundred-year history of search, deciphering and analysis*. (Questions of the history of natural science and technology. 2021. Vol. 42. No. 1), 71–88], where the period 1950–1968 is described in some detail.

[33] Russian State Archive of Social and Political History (RGASPI). F. 71. Op. 4. D. 177. L. 65–89, 98–107.

[34] RGASPI. F. 71. Op. 4. D. 177. L. 66. We will omit here other derogatory characteristics of Gumbel, as well as other scientists, which are so attractive to some authors.

[35] Mathematicians A.S. Kuzichev and Z.A. Kuzicheva were also invited to work. As evidence of the beginning of this work we will call the article Z.A. Kuzicheva, K.A. Rybnikov, *Matematika v nauchnykh issledovaniyakh K. Marksa*. (Istoriya i metodologiya estestvennykh nauk (IMEN). 1986. Vyp. 32), 3 – 13 [*Mathematics in the scientific research of K. Marx* (History and methodology of the natural sciences (IMEN). 1986. Issue 32), 3–13].

[36] Marx K., Engels F., *Collected Works*, (LW, 2010, Vol. 42), 138.

[37] Karl Marx and Mathematics. A Collection of Texts in Three Parts. Ed. by Pradip Baksi (Delhi: Aakar Books 2018), 201 – 204.