

**Marx's Mathematical Manuscripts and
the History of Their Publication**

1. Reasons and aims of Marx's mathematical studies
2. The topicality of Marx's research in mathematics
3. The Subject and Internal Logic of Mathematical Manuscripts
4. Marx's mathematics study as a non-linear cognitive process
5. Features of the epistolary form of objectification of scientific knowledge
6. Marx's note «On the Concept of the Derived Function»
7. The note «On the Differential» and its outlines
8. The addressee of Marx's mathematical notes and the addition to the note «On the Differential»
9. The central problem of the mathematical manuscripts
10. Marx's manuscripts on the history of mathematics
11. History of the publication of K. Marx's «Mathematical Manuscripts»
12. The first attempts to publish Marx's mathematical manuscripts. E.J. Gumbel's work on their systematization
13. The first publication of fragments of mathematical manuscripts by K. Marx in Russian by a team of mathematicians led by S.A. Yanovskaya
14. Work on the publication of K. Marx's book «Mathematical Manuscripts». Moscow: "Nauka". 1968.
15. Further work on the publication of the mathematical manuscripts of K. Marx

Marx's Mathematical Manuscripts and the History of Their Publication

Arithmetic was always a stumbling block for Marx. He often made mistakes in simple calculations. Marx complained to F. Engels in the letter on January, 11, 1858: «In elaborating the principles of economics I have been so damnably held up by errors in calculation that in despair I have applied myself to a rapid revision of algebra. I have never felt at home with arithmetic. But by making a detour via algebra, I shall quickly get back into the way of things»¹.

But Marx's «blitzkrieg» on mathematics failed. His study of it lasted more than 30 years. At first it was only sporadic, on an ad hoc basis, and only in the 1870s did it take on a systematic character. But as a result, only a few notes (fair copies), which set out the basic concepts of differential calculus and its elementary operations, are considered as independent works of Marx on mathematics. But these notes do not at all testify to the level of Marx's knowledge in the field of differential calculus. They were the «tip of the iceberg», these texts are based on a broad basis of mathematical studies.

¹ *Marx K., Engels F.* Collected Works. LW 2010. Vol.40. P. 244. Let us briefly recall, for the sake of completeness, the known data on the history of the creation of Marx's mathematical manuscripts.

Marx made extensive excerpts from mathematical books and textbooks, especially on differential calculus. He was possessed of deep knowledge of the differential calculus, the foundations of mathematics and its history. The problems he posed were primarily historical-mathematical and methodological in nature. So what, the mountain gave birth to a mouse, as Aesop said?

No, quite the opposite. In order to give an adequate assessment of these works of Marx on mathematics, one should not consider the two notes of Marx intended specially for Engels as the goal and result of Marx's studies in mathematics. On the contrary, one should take into account the whole complex of his works on this subject.

The assessment of the scientific significance of K. Marx's studies in mathematics is a subject of discussion among historians of mathematics and philosophers. Some authors are convinced of the innovative nature of Marx's conclusions. Others noted that the manuscripts were mostly excerpts from mathematical literature, and still others insist on the applied significance of Marx's mathematical studies in order to provide a mathematical basis for economic theory. The contradictory nature of these assessments is due to the variety of approaches to Marx's mathematical studies. One cannot help but admit that under certain political conditions the evaluation of Marx's mathematical manuscripts was sometimes influenced by the task of demonstrating their political and ideological significance.

In undertaking an examination of the significance of Marx's mathematical heritage, several problems should be highlighted. First, as in the case of natural science, there is still no sufficiently convincing answer to the question of the reasons and purposes of Marx's engagement with mathematics and, in particular, differential

calculus. The development of this problem would make it possible to determine the place of Marx's mathematical studies in his work as a whole.

Secondly, it is necessary to show how relevant the methods, conclusions and, most importantly, the subject matter of Marx's mathematical works were for their time.

Third, it is required to investigate the logic and structure of Marx's mathematical manuscripts as a whole. At the same time it is crucial to answer the question of what preparatory works, including outlines of mathematical literature and rough drafts, preceded the manuscripts, which are traditionally regarded as Marx's presentation of his vision of the basic concepts of differential calculus.

Turning to a more detailed examination of the totality of Marx's mathematical manuscripts, it is essential, fourthly, to characterize the main problems and peculiarities of Marx's treatment of them.

1. Reasons and aims of Marx's mathematical studies

Speaking about the reasons and aims of Marx's mathematical studies, we should take into account several sources of analysis of this problem. There is the evidence of reflection by the author of the manuscripts himself and by Engels, their addressee. The purpose of Marx's mathematical studies can be deduced from a consideration of both the manuscripts themselves and the reflection of his work on them in other spheres of his theoretical interests, particularly in political economy. Thus, in the manuscripts of *Capital*, one can easily find Marx's remarks showing the role he attributed to mathematics in the development of social production and political economy. We can draw conclusions about why Marx undertook the

study of mathematics by discussing the laws of development of theoretical knowledge and epistemology in general. Marx's mathematical manuscripts illustrate in the best possible way all the stages of scientific cognition: from receptive activity to the formulation of his own concept. Marx collected and analyzed various data from mathematical disciplines, defined a central category of his own research, created of plans for the further study of mathematics and, finally, attempted to write original texts.

* * *

Engels was the first biographer and researcher of Marx's work who raised the question of the reasons and purposes of Marx's mathematical studies at the motivational level. He put forward two hypotheses, which later became widespread. The first of them emphasized the utilitarian significance of these studies, namely, the necessity of studying mathematics to create the mathematical apparatus for the system of political economy to be developed. This understanding was by no means accidental.

Having passed to the study of mathematical analysis, Marx saw the prospects of its practical application not only in political economy, but also in military affairs, about what he wrote to Engels on July 6, 1863, offering him at the same time one of his books: «My spare time is now devoted to differential and integral calculus. Apropos, I have a superfluity of works on the subject and will send you one, should you wish to tackle it. I should consider it to be almost essential to your military studies. Moreover, it is a much easier branch of mathematics (so far as mere technicalities are concerned) than, say, the more advanced aspects of algebra. Save for a knowledge of the more ordinary kind of algebra and trigonometry,

no preliminary study is required except a general familiarity with conic sections»².

Nowadays it is difficult to unambiguously answer the question of what kind of book Marx intended to send to Engels. However, it is known that a year later Engels studied the textbook of Louis-Benjamin Francoeur sent to him by Marx. Engels had enough knowledge of the material to make some criticisms. Engels' attention was drawn to the section on arithmetic, where he noted the typos that Marx had not corrected. He wrote on May, 30, 1864: «I have been engrossed in the arithmetic in your Francoeur. A section you would seem pretty well to have ignored, if the failure to correct the scandalous printing errors in the figures is anything to go by. Though individual bits are quite elegant, the practical aspect of arithmetic is handled in a shockingly inept and superficial manner, being better taught at any German school. I also doubt whether it is practical to discuss things such as roots, powers, series, logarithms, etc., even at elementary level, merely in terms of numerals (without any recourse to algebra and, in fact, without presupposing an elementary knowledge of the same). Although the use of numerical examples by way of illustration may be a good idea, I should say that to limit oneself to numerals is, in this case, less conducive to clarity than simple algebraic treatment with $a+b$, precisely because the general expression is simpler and clearer in algebraic form and is something which cannot be dispensed with here. Admittedly, this particular section is really beneath the dignity of the mathematician par excellence»³.

² Marx K., Engels F. Collected Works. LW 2010. Vol. 41. P. 484.

³ Marx K., Engels F. Collected Works. LW 2010. Vol. 41. P. 532.

Not being the main occupation in the 1860s, mathematics was only sporadically present in Marx's and Engels's natural science studies. It is therefore understandable that they rarely addressed these subjects in their correspondence, sometimes only a few times a year. On November 23, 1860, Marx reported: «The only occupation that helps me maintain the necessary quietness of mind is mathematics»⁴. From his letter to Engels on May 20, 1865 we learn: «In between times, since one cannot always be writing, I am doing some differential calculus $\frac{dy}{dx}$ »⁵.

According to another version of Engels, the reason for the study of mathematics and natural science lay in the fact that, especially after 1870, the periods of Marx's creative activity were interrupted during illness, when he was able to exclusively receptive type of scientific activity. «There was another intermission after 1870, due mainly to Marx's ill health. As usual, Marx employed this time for studies; agronomics, rural relations in America and, especially, Russia, the money market and banking, and finally natural sciences such as geology and physiology, and above all independent mathematical works, form the content of the numerous excerpt notebooks of this period», so Engels wrote about the occupations of his associate⁶.

* * *

However, reducing the conclusions about the causes of mathematics to evidence of reflection, on the one hand, leads to a narrowing of the aspects of analysis, and on the other, to a distortion

⁴ Marx K., Engels F. Collected Works. LW 2010. Vol. 41. P. 216.

⁵ Marx K., Engels F. Collected Works. LW 2010. Vol. 42. P. 159.

⁶ Marx K., Engels F. Collected Works. LW 2010. Vol. 36. P. 7.

of the objective significance of Marx's mathematical works. An analysis of Marx's economic manuscripts reveals that the conditions for the study of mathematics were related to attempts to «mathematize» economic research in some way, to present the structure of political economy in a more illustrative algebraic form, to find analogies between economic and mathematical concepts.

Marx discussed one such «application» of the mathematical apparatus with his associate, the mathematically educated lawyer Samuel Moore (1830–1912). On May 31, 1873, Marx reported to Engels: «I have been telling Moore about a problem with which I have been racking my brains for some time now. However, he thinks it is insoluble, at least pro tempore, because of the many factors involved, factors which for the most part have yet to be discovered. The problem is this: you know about those graphs in which the movements of prices, discount rates, etc., etc., over the year, etc., are shown in rising and falling zigzags. I have variously attempted to analyse crises by calculating these ups and downs as irregular curves and I believed (and still believe it would be possible if the material were sufficiently studied) that I might be able to determine mathematically the principal laws governing crises. As I said, Moore thinks it cannot be done at present and I have resolved to give it up for the time being»⁷. The analysis of the mathematical manuscripts themselves, and especially of the plans of Marx's subsequent studies, gave every reason to assert that Marx did not go further than these statements. These reflections were not continued and remained apart from the main direction of Marx's mathematical studies.

⁷ Marx K., Engels F. Collected Works. LW 2010. Vol. 44. P. 504.

The exception is the presentation of the problems of the rate of profit, the rate of surplus value, etc., which is confirmed in Marx's economic manuscripts⁸.

In the process of studying this science, his initial aims underwent a certain transformation: from studies motivated by the utilitarian tasks of economic research, Marx came to realize the need to rethink the foundations of differential calculus. This meant that, preferring mathematical analysis, he set himself the task of first investigating the foundations of differential calculus and then presenting his results.

* * *

The conclusion is that Marx's work in mathematics, which lasted for almost thirty years, was a form of expression of his inner intellectual potential, as well as an additional area of his creative interests, and was not limited to the application of mathematics to political economy or simply a way of distracting himself from his ailments. The *reasons* for Marx's interest in mathematics and an incentive to start studying this science lay in the field of attempts to mathematize political economy. The *conditions* for studying mathematics were dictated by external circumstances, such as the need to alternate productive and receptive scientific activities. These factors, however, did not constitute the ultimate *goal* of the

⁸ See: *Marx K. Formeln und Berechnungen zu Mehrwert- und Profitrate // MEGA². Bd. II/14. S. 3–7; Derselbe. Rate des Mehrwerts und Profitrate // Ebenda. S. 8–18; Derselbe. Mehrwertrate und Profitrate mathematisch behandelt // Ebenda. S. 19–150.*

mathematics studies. Based on the study of the manuscripts themselves, we can conclude that the *aim* was to try to justify the method of differential calculus and to interpret its basic concepts and theorems. This is related to the philosophical significance of the mathematical manuscripts, which lies not in the relationship between mathematics and political economy, but in the methodological justification of differential calculus, in development of the methodology of scientific knowledge in general.

2. The topicality of Marx's research in mathematics

Marx's studies of mathematics in general and differential calculus in particular took place between the late 1850s and the early 1880s. It seems appropriate from the outset to give a brief sketch of processes in the field of differential calculus that attracted Marx's attention and were constantly at the centre of his scientific interests in order to answer the question about the relevance of his studies, about their correspondence to the tasks of the time, as well as to evaluate more accurately the problematics of Marx's manuscripts and the new things he contributed to the understanding of the issues under consideration.

The history of differential calculus goes back to the second half of the 17th century, to the works of Newton and Leibnitz. From spring to fall of 1665 I. Newton developed the principles of analyzing infinitesimal quantities (fluxions). His method was based on the decomposition of a function into a power series and gave the concept of differentiation. In addition, his research allowed him to

define integration as a function inverse to differentiation and to prove the interdependent nature of differential and integral calculus.

Confidence in the possibility of decomposition of any function into a power series served as a reason for Newton to consider differential calculus universal. The results of his research were formulated in his work «De analysi per aequationes numero terminorum infinitas» (1669). Newton began to use the specific symbolism of calculus from the end of 1691⁹. His work «The method of fluxions and infinite series: with its application to the geometry of curve-lines», written in 1670–1671, was devoted to the same problems, but it was published only later – in 1736.

Independent of Newton, but ten years later Leibnitz came to a similar discovery. He had developed the principles and symbolism of differential and integral calculus, which were published in «Nova methodus pro maximis et minimis itemque tangentibus, quae nec fractas nec irrationales quantitates moratur et singulare pro illis calculi genus» (1684). The results of Leibnitz's research in a new field of mathematic knowledge were made public, when similar developments by Newton were still in manuscript. Newton and Leibnitz's acquaintance with each other and their correspondence gave Newton reason to doubt the independence of Leibnitz's discovery, which eventually led to their long spat.

Along with the method of fluxions, Newton developed the calculus of finite differences, i.e. the calculus of finite increments of

⁹ Newton first openly formulated the principles of his version of differential calculus in letters to John Wallis (1616–1703), and with the author's permission, Wallis published these letters in a reprint of his «Treatise of Algebra» (1693).

arguments and functions, the algorithm for calculating which is similar to the algorithm of differentiation.

The works of Brook Taylor (1685–1731), in particular his work «Methodus Incrementorum Directa & Inversa» published in 1715, were also in this area. Taylor's name is usually associated with the Taylor series, with the theorem of his name on the decomposition of a function into a power series¹⁰. It is characteristic that Taylor's series is currently given in the notations formulated by the French mathematician Joseph-Louis de Lagrange. In 1742, Colin MacLaurin (1698–1746) formulated a theorem on the decomposition of a function with indefinite coefficients into a series and found them. The result became known as MacLaurin's theorem.

The emergence of a new direction in mathematics, which was from the very beginning connected with calculations in the field of mechanics, was at first not supported by philosophical foundations. This gave rise to justifiable criticism of differential calculus. George Berkeley in a number of works, and in particular in his «A Treatise Concerning the Principles of Human Knowledge» (1710) addressed to Edmond Halley, reproached the founders of the calculus for the vagueness of the basic concepts.

The justification of the analysis of infinitesimal quantities was carried out practically in parallel with the development of its basic theorems. MacLaurin formulated the method of exhaustion, in the development of which he relied on the sensationalist methodology of John Locke (1632–1704), the main principle of which was the requirement to find in experience all analogs for

¹⁰ However, there is speculation that this series was known to Newton as well as to I. Bernoulli and G. Leibniz.

mathematical concepts. To defend Newton's doctrine against Berkeley, MacLaurin devoted his «A Treatise of Fluxions», published in 1742. His idea was supported by John Landen (1719–1790) in his work «The Residual Analysis» (1764). Here Landen avoided the difficulty of calculating infinitesimal quantities by renaming variables.

The justification of differential calculus was continued by Leonhard Euler (1707–1783), who proceeded from the identification of infinitesimal quantities with zero. He outlined the principles of this calculus, which he called the calculus of zeros, in his work «Institutiones calculi differentialis (Foundations of differential calculus)» (Foundations of differential calculus) (1755). Euler considered it permissible to operate with zeros as with ordinary natural numbers, up to division by zero¹¹. In differential calculus, the expression $\frac{0}{0}$ according to Euler, was not an uncertainty, but to avoid confusion, this expression was replaced by $\frac{\Delta y}{\Delta x}$. Euler himself did not always adhere to this assumption. Moreover, it did not find much support among mathematicians. In a series of articles in the famous «Encyclopédie», the French mathematician D'Alembert (Jean-Baptiste le Rond) (1717–1783) proposed his justification of differential analysis by introducing the concept of limit.

Attempts to justify differential calculus were made by Joseph-Louis de Lagrange (1736–1813). It was Lagrange who set himself the goal of presenting it without using the notion of infinitesimal quantity, as well as without the notions of limit and derivatives of

¹¹ This position does not contradict the modern ideas about the ring of real numbers.

functions. For this purpose, he formulated the theory of analytic functions, the task of which was to solve the main problems of analysis. From Newton's time until the second half of the 18th century, functions were introduced to facilitate the operation with powers by series. It was assumed that any analytic expression could be decomposed into a series. This concept was developed by Taylor, MacLaurin, Lagrange.

Lagrange tried to prove what his predecessors silently accepted, namely, that the resulting series contains only integer and positive increments of the argument. The starting point for Lagrange was the Taylor series. Lagrange proved that an arbitrary function can be decomposed into a series algebraically. Lagrange's work made it possible to discard Newton's geometric approach to the justification of calculus. However, for all the progress of this approach, its unsatisfactory feature was that Lagrange did not take into account the convergence of series.

Lagrange was one of the mathematicians who linked eighteenth- and nineteenth-century mathematics. Without pretending to analyze this vast field of knowledge in depth, we will name only a few names that determined the development of mathematics in the 19th century, against the background of which Marx's research took place. The principal role here was played by the work of the French mathematician Augustin-Louis Cauchy (1789–1857), who is considered to have given the foundation of analysis that is now generally accepted¹². In his calculations, Cauchy relied on Lagrange's

¹² Charles Hermite (1822–1901), after Cauchy's death, became the leading representative of Calculus in France.

research, using his notations, the results of the theory of real functions, but without borrowing his algebraic treatment of differential calculus. The mention of the background of Cauchy, who was also a contemporary of Marx, is all the more important because Marx also drew very extensively on the results presented by Lagrange. However, unlike Cauchy, Marx, who was predominantly interested in the method of differential calculus, relied on Lagrange's method as the basis of his own calculations.

However, the development of mathematics in the 19th century was by no means limited to the problems of differential calculus. The works of Évariste Galois (1811–1832) and the already mentioned Cauchy developed the ideas of Lagrange and Paolo Ruffini (1765–1822) in the field of group theory. Niels Henrik Abel (1802–1829) developed problems of integration of algebraic functions. The studies of Carl Gustav Jacob Jacobi (1804–1851) and William Rowan Hamilton (1805–1865) were devoted to mathematical aspects of dynamics. And the work of Johann Peter Gustav Lejeune Dirichlet (1805–1859) of that time is a significant contribution to number theory.

The 1850s also saw Bernhard Riemann's (1826–1866) research on the theory of complex variable and linear differential equations with algebraic variables. Riemann's development of the problems of topology was in close connection with his work in the theory of functions of a complex variable. The research of Karl Weierstrass (1815–1897) was also in the same field. The works of Leopold Kronecker (1823–1891) in the 1870–80s laid the foundation for such a direction in the justification of mathematics as intuitionism. The works of Georg Cantor (1845–1918) were also devoted to the problems of justification of mathematics. We should

also mention studies in geometry by János Bolyai (1802–1860) and Nikolai Ivanovich Lobachevsky (1792–1856).

The English school of mathematics stood somewhat apart. In 1812, Cambridge mathematicians under the leadership of Robert Woodhouse (1773–1827) began to propagandize the achievements of continental science, the isolated development of which from the corresponding research in England was connected with fundamental disagreements on the question of priority in the discovery of differential calculus. In 1816, the work of the French mathematician Silvestre François Lacroix «*Traité du Calcul Différentiel et du Calcul Intégral*» (1797) was published in English under the heading «*An Elementary Treatise on the Differential and Integral Calculus*». The main directions of research of English mathematicians were algebra and its application to geometry.

In the works of William Rowan Hamilton (1805–1865) a rigorous construction of the algebra of complex numbers was given. He is also considered to be the founder of Vector calculus. George Boole (1815–1864) laid the foundations of the algebra of logic, which reached its peak in the work of Alfred North Whitehead and Bertrand Russell «*Principia Mathematica*» (1910, 1912, 1913) Thus, the specialization of mathematics grew in the 19th century. Marius Sophus Lie (1842–1899) systematically studied groups of continuous transformations.

Against the background of these studies, Marx turned not to the problems of differential calculus that were contemporary to him, although he used the latest textbooks. The reason for this is that, to a large extent, the issues he raised were mainly historical in nature. The same is evidenced by the names of mathematicians whose works were first of all comprehended in mathematical manuscripts: Newton

(1642–1727), Leibnitz (1648–1716), Taylor (1685–1731), MacLaurin (1698–1746), Landen (1719–1790), D'Alembert (1717–1783), Euler (1707–1783), Lagrange (1736–1813)¹³. However, the history of mathematics did not interest Marx per se; his interest in it had its own logic, connected, as already mentioned, with the attempt to give a new justification for the method of differential calculus.

3. The Subject and Internal Logic of Mathematical Manuscripts

Alongside economic manuscripts and excerpts on a wide variety of problems, Marx's mathematical manuscripts represent a broad field for studying the methodology of the formation of theoretical views. The point is that all stages and levels of scientific research – both the receptive type of theoretical activity and its productive phase – were represented here in a concentrated form. A significant place in the mathematical manuscripts was occupied by extracts and numerous excerpts from contemporary educational mathematical literature, as well as treatises on mathematics by the creators of differential calculus. The study of the subject of differential calculus led to the identification of several problems, which were then prioritized. The extracts became more unidirectional and the selection of literature focused around a few topics. This was also facilitated by the first attempts to express his attitude to the problems under study.

On this basis, Marx compiled not just excerpts, but second-level excerpts in which he analyzed his own extracts. These were

¹³ The names of Poisson, Laplace, and Moigno were mentioned by Marx without any comment.

accompanied by comments and conclusions, which became the basis for essays on the basic concepts of differential calculus.

These objective logical stages of research often overlapped in this particular research process. Thus, extracts often followed independent sketches, with the problems of the latter giving rise to a new appeal to mathematical literature, they were a kind of criteria for selecting material and evaluating predecessors. Unlike Marx's other research in the field of natural science (geology, chemistry, etc.), his study of mathematics reached the level where he could present his own vision of the problems.

The objective course of the analysis was closely connected with Marxian peculiarities of the research. Thus, based on the concept of the unity of natural and social knowledge and, consequently, of research methods in natural science and in social sciences, Marx tried to apply to the study of mathematics the same methodological – dialectical – techniques that he had used in philosophical and economic works. First of all, it was a special consideration of what Marx contemplated to be the central problem of differential calculus – Taylor's theorem. Marx esteemed it from the point of view of the unity and struggle of opposites (the connection between algebraic and differential methods). The main logic of research in mathematics was subordinated to the question of the relation between algebraic and differential methods.

Marx narrowed down the subject of his research to the basic concepts of differential calculus, such as the derivative, the

differential coefficient, various methods of proving Taylor's theorem etc., which he studied throughout his work in this field¹⁴.

Although differential calculus (its history and the justification of its basic concepts), which included the elucidation of the physical meaning of the basic differential operations, was a major subject of study for Marx, it cannot be said that he confined himself to these issues. In his extensive manuscripts on mathematics, there were a number of problems that were studied and treated with varying degrees of thoroughness. In addition to differential calculus, Marx studied commercial arithmetic, geometry, trigonometry, algebra, and so on. These sections of mathematics formed the background against which the direct subject of research – the method and basic concepts of differential calculus – was discussed. These problems occupied a central place in all manuscripts and also constituted the nodal points of Marx's planned presentation of the elementary foundations of differential calculus.

Thus, among the wealth of mathematical problems, Marx chose differential calculus and gave a generalized characterization of its central problem, the proof of Taylor's theorem and its special case, MacLaurin's theorem. They aroused his interest in connection with the analysis of differentiation methods and the relation between differential and algebraic methods. In doing so, the history of mathematics served as a necessary background. Historical-

¹⁴ Amsterdam International Institute of Social History (IISG). A 104. See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 41–52, 53–59, 64–86, 213 etc. Marx expounded Taylor's theorem and MacLaurin's theorem as its special case in different manuscripts. See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 214–230, 260–301.

mathematical and philosophical reminiscences were present in most of the manuscripts on mathematics.

As already mentioned, Marx paid special attention to Lagrange's method of algebraic differentiation. In presenting his vision of this method, Marx limited himself to the two basic concepts of the derivative and the differential, but did not abandon his intention to continue to explore further issues including a historical overview of differential calculus¹⁵.

The different ways of proving Taylor's theorem were conditioned by different historical methods of differentiation. Although calculations were present in Marx's work, there is no reason to claim that they were the purpose of his mathematics study. Analyzing the methods of proving Taylor's theorem, Marx attempted to develop his own method of differentiation. He studied the relationship between differential and algebraic methods and attempted to determine their reciprocity. As a result, he formulated the methodological principle of *method reversal (Umschlag in der Methode)*¹⁶, which, although not widely represented in Marx's manuscripts, was nevertheless characterised by him as a fundamentally new aspect of his mathematical work.

4. Marx's mathematics study as a non-linear cognitive process

¹⁵ See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 61.

¹⁶ There are different modes of translation of concepts *Umschlag in der Methode* into English : inversion of method, method reversal, reversal of the method.

Mathematical calculations had been present in Marx's manuscripts since the 1840s. However, at that time they did not have the character of a coherent statement and occurred sporadically. As an example, one can take notebooks with extracts on political economy, which contained abstracts of works by Schütz, List, Ricardo, and Quesnay as well as the manuscript «Grundrisse» of 1857–1858, where one can find mathematical calculations¹⁷. It was only in the 1860-80s that the extracts acquired the character of a systematic study of mathematics.

However, for a complete picture of mathematical research and Marx's own ideas about the essence of differential calculus, it is necessary to consider the initial period of Marx's mathematical studies. Thus, in September–October 1851, he made an abstract of Poppe's book «History of Mathematics» (Poppe «Geschichte der Mathematik seit der ältesten bis auf die neueste Zeit» (Tübingen, 1828)¹⁸. It contained extracts from the introduction to the book, treating of the mathematics of antiquity and the Middle Ages. This was followed by extracts from the first part of the book on the history of arithmetic, the invention of logarithmic tables by Neper and Briggs. They were continued by brief extracts on the history of geometry, trigonometry, algebra, differential calculus, and mechanics. Extracts from Poppe's book were also contained in an excerpt notebook entitled «Diversa» (1867–69)¹⁹.

¹⁷ See: MEGA². Bd. IV/2. S. 503 – 505. Bd. IV/5. S. 281–288. All mathematical entries in Marx's excerpt notebooks from the 1840s date back to 1858.

¹⁸ See: MEGA². Bd. IV/10.

¹⁹ See *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P 112.

Marx's extracts from the first volume of Sauri's book «Cours Complet De Mathématiques» (1778)²⁰ belong to the early 1860s. Marx's attention was drawn to the sections «On Trigonometry», «On Solving Triangles», and «On Solving Oblique Triangles». In addition, the outline had Summary of formulas on trigonometry, calculation of a number of trigonometric functions, transformation of formulas to a form convenient for logarithmisation. These formulas were of an applied nature, as they could be used to measure the width of a river, the height of a tower or a mountain, the distance between inaccessible places, and were also used to maps shooting. At the same time, Marx appreciated the importance of trigonometry for understanding differential calculus.

Marx's first notes on commercial arithmetic were written in the late 1860s. In an notebook, dated «1869. First notebook», there was an outline of Goshen's book «Theory of International Exchange» and the section on bills of exchange in the 1859 edition of Feller and Oderman's «Complete Course of Commercial Arithmetic», which was adjacent to it in terms of subject matter. At the end of the extracts, Marx gave the table of contents of the different parts of these books. In the same year, but in another notebook, Marx continued this outline on the problems of commercial arithmetic²¹.

Marx's studies of applied problems of mathematics – the application of algebra to geometry – i.e. the theory of conic sections, were evidenced by the extracts from J. Hymers' book «Treatise on Conic Sections», 1845 and the outline of the sections of the second

²⁰ See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 115.

²¹ Amsterdam International Institute of Social History (IISG). B 113, B 114. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 116–118.

volume of the mentioned treatise by Sauri, which were close to them in content²². These extracts were entitled «Conic Sections». They indicate Marx's interest in the applications of algebra to geometry and serve as a mediating link between his studies of applied mathematics and the study of differential calculus. It should not be forgotten that differential calculus itself historically arose from the need to solve applied problems of mathematics. Thus, Marx's transition to the study of this section of mathematics coincided with the historical development of this science.

Marx's first note on differential calculus was an outline of the first sections of J.L. Boucharlat's book «An Elementary Treatise on the Differential and Integral Calculus» (Cambridge; London. 1828. Paragraphs 5–18)²³. Marx knew the basics of differential calculus from the above-mentioned work of Sauri, with which he constantly compared this textbook. Marx turned to Sauri's work, or more precisely to its third volume, for information on the method of finite differences. However, these problems were combined at this time with the study of commercial arithmetic in Feller and Oderman's book. In a notebook dated «II. Begun March 1878», Marx made extracts on the calculus of bills of exchange and other mathematical problems of commerce, interspersed with extracts on arithmetic and geometric progressions and finite differences from Sauri's book²⁴.

The next large outline on differential calculus was made by Marx also from Sauri's treaty, from Newton's «De analysi per

²² See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 119.

²³ See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 119–121.

²⁴ Amsterdam International Institute of Social History (IISG). B 141. See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 122.

aequationeses numero terminorum infinitas» (1669) and from the textbooks of Boucharlat and Hind, in which the starting point was Lagrange's algebraic calculus²⁵. Here Marx began to take another look at the sections of Boucharlat's work devoted to Lagrange's method. In this period, Marx was not yet as critical of this method as he had become during the development of his own conception and tried to understand his views. Along with extracts, Marx commented on the texts he read, thinking about the difference between the expressions $\frac{0}{0}$ in algebra and $\frac{dy}{dx}$ in differential calculus.

Marx connected the consideration of Taylor's theorem with the analysis of Lagrange's method of differentiation, reasoning about the essence of the introduction of the symbolism of differential calculus. In this excerpt, Marx explained the reasons for his interest in this theorem by the fact that «theorem of Taylor ... under certain constraints, may be viewed, in a determinate sense, as the basis of differential calculus»²⁶.

Over time, Marx's own comments began to appear more and more often in his excerpts, which served as a basis for his further research. While studying the problems of mathematics, Marx did not cease to be a philosopher. Approaching these problems from the perspective of dialectics, he remarked about mathematical operations that «in the negation is retained that *qualitative relation*, of which

²⁵ See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 123–150. The dating of this notebook was carried out taking into account the bibliographic entries on pp. On the last two pages of this manuscript there are also notes «Continuation in another notebook (III, following Kaufman II) (Last page)». The notebook III referred to in the notation was written after March 1878.

²⁶ *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 134.

this transformation is the negation»²⁷. The next manuscript to be mentioned²⁸, although it began with an outline of Feller and Oderman's *Commercial Arithmetic*, was mainly devoted to differential calculus. Here Marx read the introduction to the first volume of Lacroix's textbook «*Traité du calcul différentiel et du calcul intégral*» Paris 1810–1819), namely those passages dealing with the concepts of function, series, and limit. This was followed by extracts from the English and French editions of Boucharlat's work, where the method of indeterminate coefficients and sequential differentiation, MacLaurin's theorem, differentiation of logarithmic and trigonometric functions were discussed²⁹. At the same time, Marx clearly favored Lagrange's method of differentiation: «The importance of this method appears in the latter analytical operations, and not in the initial ones»³⁰.

* * *

A large complex of works on algebra comprised the notebooks with excerpts «Algebra I» and «Algebra I». As in the

²⁷ Marx K. *Mathematical Manuscripts*. Calcutta. 1994 P. 138.

²⁸ Marx K. *Mathematical Manuscripts*. Calcutta. 1994. P. 151–166.

²⁹ In this excerpt Marx, relying on mathematics manuals, compared the theorems of Taylor and McLaurin, noting that they proceed from different assumptions: McLaurin takes as a basis the function itself, Taylor – the increment of the function. Marx concluded that McLaurin's theorem can be derived from Taylor's theorem. Marx then analyzed the tangent problem and two methods of differentiation. The first method was the method of limits, which was based on the method of finite differences and did not go beyond algebra. Here the differential was understood as the minimum value of the difference. Marx also had at his disposal the textbook of the British mathematician Hall Th. G. «*The elements of algebra*» (3rd ed. Cambridge, 1850), from paragraphs 95–96 of which Marx wrote out information about the expansion of functions of several variables into Taylor series. In addition, here he made extracts from volume III of Sauri's «*Cours Complet De Mathématiques*» (1778) of information about differentiation of a product, and finally, from paragraphs 255–260 of Boucharlat's textbook – about cases of inapplicability of Taylor's formula.

³⁰ Marx K. *Mathematical Manuscripts*. Calcutta. 1994. P. 166.

excerpts of later times, there were Marx's own reflections on the concept of function, on the connection between differential calculus and algebra, i.e. on the problems which in the future became the subject of special studies. The notebook entitled «Algebra I»³¹ belonged to the second half of the 1870s. In this excerpt, Marx gave his own reasoning, which testified to a sufficiently high degree of mastery of the problems under consideration. He analyzed the processes occurring in the different parts of the differential equation, which was also one of the main themes of his own considerations in his letters on mathematics addressed to Engels³².

In the notebook «Algebra II»³³ Marx continued his study of Lacroix's work «Éléments d'algèbre», 11-me ed., Paris, 1815, as well as MacLaurin's «Treatise of Algebra» (1796) and others. Here were collected notes on problems of proportions and progressions, exponents and logarithms, and, in addition, money interest. The second part of the abstracts was a systematization of material from various sources on problems of sets. This was followed by a

³¹ Amsterdam International Institute of Social History (IISG). A 100. See : *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 167 – 184.

³² These extracts were followed by the part «III. Rudiments of infinite serieses», which dealt with the approximate computation of roots, and then with infinite series proper. In part «IV. On the concept of function» also drew on the works of Lacroix and Euler. The last part of the notebook «V. Elimimlfion of unknowns form equations of power greater then one» contained extracts on equations of the first degree, finding the greatest common divisor, finding rational and simple roots of numerical equations, and approximate solution of numerical equations. In this part of the outline, Marx used, among other things, MacLaurin's «Treatise on Algebra», on the basis of which he argued about the relationship between algebra and differential calculus.

³³ *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 185 – 210.

discussion of the binomial theorem, to which Marx devoted an entire section³⁴.

* * *

It is difficult to claim a complete description of Marx's vast handwritten legacy of his mathematical works. But any study of Marx's mathematical excerpts would be incomplete without mentioning the so-called second-order excerpts. This is where the material learnt at the previous stage of mathematics was systematised. The extracts made earlier were grouped around a few topics that were to play the role of the main subject matter to be dealt with later. This mode of research was similar to the process of developing the foundations of political economy, moving from mere note-taking, through the summarisation of one's own notes, to the outline of one's own concept and finally to independent work.

It cannot be said that second-order excerpts, as a result of the transition from a purely receptive activity to a productive one and thus to the formation of an original concept, are inherent in any research. They are likely to be a characteristic feature specific to Marx. Nevertheless, their study does not lose its importance, as it helps to identify the structural elements of any course of cognition at the stage immediately preceding the transition to new knowledge.

The second-order excerpts in mathematics were located at the boundary between new and old knowledge, where the evaluation and

³⁴ Here the works of L. Euler, C. MacLaurin, Th.G. Hall and others were summarised. On this basis, the empirical origin of the theorem was elucidated, its proof for a particular case was given and the conditions for its generalisation were given. This outline was continued in another algebra notebook. Here the outline of chapter V of MacLaurin's «Algebra» was finished and chapter VI of the same book was outlined. The next manuscript contained the outline of J. Hinde's «The elements of plane and spherical trigonometry» (Cambridge. 1837) (See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 211–213).

systematisation of acquired knowledge, which represented the previous stage of the development of science, took place. It was here that Marx's own conception and vision of problems were born. In contrast to the first-order excerpts, the selection of material here was less subject to chance and, on the contrary, more and more purposeful.

On the contrary, in the first-order excerpts, Marx involuntarily followed the historical development of science, repeating at the individual level the transition to the problems of differential calculus, based on the needs of solving practical problems. In other words, the transition from the concrete to the abstract was underway. In the second-order excerpts, the logical basic categories of the methodology of differential calculus were identified, which could provide a source for the further development of one's own concept. It was for this purpose that the systematisation of the material studied was undertaken. In Marx's mathematical excerpts, the systematisation was made around the problems of the relation between the algebraic and differential methods, around the basic problems of differential calculus, as well as Taylor's and Maclaurin's theorems.

In the excerpts of this group one can find attempts to present one's own vision of the problems of systematised knowledge. It happened that the first experiments of independent exposition followed a false path. An example of such a beginning, which was not followed by a continuation, was Marx's study on the concept of function. Although this notion was of the most abstract character, it was not of fundamental importance for the purposes of his subsequent exposition, i.e. the development of his own vision of the problems of differential calculus did not then follow the path of

investigating the nature of the function. Having returned to the systematisation of the material, Marx continued to collect material on the proof of Taylor's theorem.

One of Marx's first attempts to analyze the material he had learned, where he systematized it, was an extensive manuscript on Taylor's theorem³⁵. Here Marx referred to his excerpt notebook «Algebra I». He evaluated the binomial theorem, the development of the general theory of equations, combinatorics, trigonometry, the foundations of differential calculus³⁶.

In the manuscript «Taylor's Theorem, MacLaurin's Theorem and Lagrangian Theorem of Derived Functions» Marx elucidated the background of Taylor's theorem³⁷. This theorem became, in Marx's estimation, the basis of differential calculus. It should also be mentioned that in the excerpt notebook «Algebra I» Marx had already devoted a whole section to Newton's binomial theorem, where he considered the empirical origin of this theorem and analyzed its proof on a concrete example.

The manuscript «Taylor's Theorem»³⁸ dealt with cases where Taylor's theorem is inapplicable. «this theorem *is* the binomial theorem with *whole and positive exponents*, translated into

³⁵ Amsterdam International Institute of Social History (IISG). A 104. See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 214–230.

³⁶ Thus, the significance of the binomial theorem for algebra, according to Marx, is equal to the significance of Taylor's theorem for differential calculus: «The *binomial theorem* (which can be extended to the *polynomials*) is the greatest discovery of algebra proper. Not only did the solution of the *equations* of determinate power, as it happened earlier, become possible with the help of this theorem, but also the general theory of *equations*» (*Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 229).

³⁷ Amsterdam International Institute of Social History (IISG). A 104. See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 231–236.

³⁸ Amsterdam International Institute of Social History (IISG). A 104. See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 93–94, 264–301.

differential language. Where these conditions are not fulfilled, where *Taylor's theorem is not applicable*, that is, there enter what are called in differential calculus the '*failures*' of this theorem»³⁹. The analysis of the theorem gave Marx the basis for introducing the notion of operational formula, which he created in the course of formulating his own method of differentiation, different from those already available in mathematics.

In the next manuscript⁴⁰ Taylor's theorem was treated on the basis of the English edition of Boucharlat's textbook, which did not present its general form, but only a special case. In addition, a historical background of the original methods of its proof was given here. Finally, Marx referred to the proof of the theorem by the French mathematician Poisson in Th.G. Hall's textbook «A treatise on the differential and integral calculus» (London, 1852).

Thus, even a brief review of Marx's excerpts on mathematics shows that a substantial part of the manuscripts consisted of extensive extracts on various problems of mathematics. At the same time, it should be noted that Marx returned to the same sources more than once, sometimes outlining them twice. The reasons for this phenomenon can be seen in the complexity of comprehension of the material under study and the consequent need to understand the essence of the problems. The need to understand mathematics thoroughly was reflected in the fact that Marx did not just write out information from the sources he studied, but often reproduced the

³⁹ Amsterdam International Institute of Social History (IISG). B 114, B 114. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 116.

⁴⁰ Amsterdam International Institute of Social History (IISG). A 104. See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 261–263.

intermediary links in the proofs that the authors had presented only in an extremely schematic form.

The absence of references to the sources excerpted in the vast majority of cases and the absence of clear boundaries between the excerpts and the author's own reflections on the readings, often in line with the author's reasoning, are indicative of the fact that the excerpts were not made for the purpose of further citation. The study of the excerpts allows us to understand how Marx developed his own assessments, which were then used as the basis for the conclusions he formulated in his independent works. These assessments were born in the process of systematizing the material under study, primarily around Taylor's theorem. Here the necessity of finding out the correlation between algebraic and proper differential methods of mathematical proofs was discovered. The study of this question advanced to such an extent that it allowed Marx to penetrate into the essence of the correlation between these methods already in the presentation of the basic concepts of differential calculus.

5. Features of the epistolary form of objectification of scientific knowledge

One of the most important (although mainly historical) forms of objectification of scientific knowledge, in other words, its presentation and dissemination, has been and remains scientific correspondence. The epistolary genre has developed in science since ancient times. Letters of Spinoza, Leibnitz, Kant and many others are a perfect confirmation of this. The problem of the addressee of knowledge was posed a long time ago and runs through many

philosophical and not only philosophical systems. In general terms, this is the problem of intersubjectivity of any knowledge.

Mathematics was an area of Marx's scientific research, about which we can draw conclusions by studying his correspondence.

The peculiarity of mathematical notes, unlike special works, was that they not only treated certain scientific problems, reflecting the level of their development by the author, but the presentation itself was oriented to the addressee to a greater extent than in other cases. Marx's mathematical notes were intended primarily for Engels, and in doing so the author took into account the level of training and interests of his addressee⁴¹.

The first attempt of this kind of exposition was in the early 1860s. Marx felt confident enough in mathematics to introduce Engels in one of his letters to the essence of differentiation. This was the immediate occasion for the first mathematical note in 1863. In it, Marx wrote: «The whole of differential calculus arose originally from the task of drawing tangents through any point on any curve»⁴².

Giving a geometrical illustration of the tangent task, Marx operated with the concepts of increment, differential, general differential equation. At the same time, he did not yet touch upon the notion of infinitesimal value, which became later the subject of special study, and he defined the notion of differential as «which represents an infinitely small increment of y »⁴³. All this suggests that in the early 1860s Marx was predominantly concerned with the

⁴¹ Marx took into account the peculiarities of perception of his works even by readers of different nationalities (see: *Antonova I. K.* New moments of the history of Marx's work on the French edition of the first volume of «Capital» // *Beitrag zur Marx-Engels-Forschung*. Berlin. 1987. S. 67–74).

⁴² *Marx K., Engels F.* Collected works. LW 2010. Vol. 42. P. 209.

⁴³ *Marx K., Engels F.* Collected works. LW 2010. Vol. 42 P. 210.

technique of differentiation, without setting the task of substantiating the differential method itself, without formulating concepts and without thinking about the method of differentiation.

It was only in the late 1870s and early 1880s that his attention began to be drawn to the justification and method of the differential calculus, and this relatively early work of Marx was the prototype of the subsequent notes to Engels on the concepts of the derived function and differential. Marx's mathematical correspondence of the 1870s and 1880s consists of some small notes, first of all «On the Concept of the Derived Function» and «On the Differential», and drafts and additions to them written in 1881.

6. Marx's note «On the Concept of the Derived Function»

On August 18, 1881, in a letter to Marx, Engels reported: «Yesterday, then, I at last plucked up the courage to make a thorough study of your mathematical mss. without any reference to manuals and was glad to find I had no need of them. I offer you my congratulations. The thing is so crystal clear that one can only marvel at the obstinacy with which mathematicians insist on shrouding it in mystery. But that is what comes of those gentry's one-sided mentality. To write firmly and categorically $\frac{dy}{dx} = \frac{0}{0}$ could never enter their heads. And yet it is obvious that $\frac{dy}{dx}$ can only be the pure expression of a process undergone by x and y when the last trace of the terms x and y has disappeared and all that remains is the

expression, free from all quantity, of the process of variation they are undergoing»⁴⁴. There were the first and second note treating of the concepts of derived function and differential. This letter serves as the basis for the tentative dating of Marx's manuscripts on the derived function and the differential – no later than August 1881. Among other things, we can conclude from this letter that Engels himself had already advanced to such an extent in the study of mathematics that he was able to evaluate Marx's contribution to the solution of this problem on the basis of other works known to him.

Marx's manuscript «On the Concept of the Derived Function» was not titled by the author and received this title only when it was prepared for printing by publishers of mathematical manuscripts. It has come down to us in an envelope bearing the name of its addressee «For Fred». It had drafts. One of the drafts was a small manuscript containing a list of formulas illustrating the original concepts of differential calculus. These formulas had been written out by Marx from the first two paragraphs of J. Hind's textbook «The Principles of the Differential Calculus, with its application to Curves and Curve Surfaces» (Cambridge, 1831), where the author cited them in connection with the definitions and initial consideration of the concept of the differential of elementary algebraic functions, particularly of one variable.

In addition, there was a direct draft of the manuscript itself «On the Concept of the Derived Function»⁴⁵. A comparison of it with the final version shows that they coincide in principle. Therefore, it

⁴⁴ *Marx K., Engels F.* Collected works. LW 2010. Vol. 42. P. 130.

⁴⁵ Amsterdam International Institute of Social History (IISG). A 104. See: *Marx K. Mathematical Manuscripts.* Calcutta. 1994. P. 249–250.

makes sense to consider the main problems treated in both manuscripts and to fix what was the difference between them, in particular, how and in what direction Marx's presentation of the derived function developed.

At the outset, let us stipulate that, in analyzing derived functions, Marx chose some of the functions given in the already mentioned Hind's textbook as the subject of direct consideration. This, however, did not mean that Marx's manuscript was a direct borrowing from Hind. Unlike the English mathematician, Marx was more interested in the philosophical and mathematical justification of differential calculus.

The first thing Marx noticed was the meaning of the concept of limit transition, in other words, the transition from the increment of a function and its argument to their original values. In contrast to the mathematical literature, where the concepts of limit, limit transition, etc. were introduced beforehand, Marx immediately went on to interpret the essence of the differential operation. He believed that as a result of this operation, the function and its argument return to their original values. «Thus, at first the postulation of a difference, and then its inverse removal will lead literally, to *nothing*. The entire difficulty in understanding the differential operation (as in that of any *negation of negation* whatever) lies precisely in seeing *how* it differs from such a simple procedure, and thus leads to valid results»⁴⁶. This statement alone shows that Marx approached the problem of differential calculus first of all as a philosopher and not as a mathematician, preferring to find a way out of the problem by

⁴⁶ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 19.

referring to a more general philosophical concept, in this case the most general law of dialectics, treated as one of its axioms and therefore not in need of proof.

Then Marx introduced the concepts of the the *preliminary* derivative of x and the *final* derivative, the limit of the ratio of differentials. Marx noted that $\frac{dy}{dx}$ is the starting point of differentials of higher degrees and «a symbol of some real process»⁴⁷, and he understood it as a set of mathematical operations to be performed to find derivatives and differentials. Marx once again demonstrated his considerations on the example of finding the derivative of the function $y = ax^m$.

Comparing Marx's presentation of the concept of the derived function with that accepted in the literature, we can conclude that Marx's approach to the problem had its own specificity. This specificity consisted in clarifying the relationship between the preliminary and final derivative. Marx made active use of the arsenal of concepts of differential calculus, sometimes without defining them.

In addition, he introduced a number of his own concepts, which, in his opinion, should supplement the traditional ones or simplify the presentation.

Marx's work on the draft manuscript and the preparation of the final version was to clarify these new notions, to concretize them. In it, Marx made the notion of the preliminary derivative more explicit, adding the word «preliminary» whenever it was mentioned

⁴⁷ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 23.

and where in the draft it was simply «derivative». The process of finding the derivative has also been shown more concretely here. It does not omit the intermediary links of transformations, which was sometimes the case in the draft.

7. The note «On the Differential» and its outlines

Marx's second independent manuscript on mathematics, addressed to Engels, was the note «On the Differential». Despite the small size of the work «On the Differential» (it took up only 13 small manuscript pages), this essay was central central to this stage to Marx's research on differential calculus. Note «On the Differential» was preceded by solid preparatory work. The first of its outlines can be considered to be that manuscript, which was one of the second-order excerpts, where Marx not only made extracts from the treatises of Sauri, Boucharlat, etc., but also commented on them.

The subject of the note «On the Differential» and its drafts was not even the concept of the differential itself, but rather the method, the process of differentiation of functions and, in this connection, the formulation of the features of the new method of differentiation that Marx claimed. Thus, he built on the results achieved by the mathematics of his time, using in his work the concepts of derivative, differential particle, etc. In addition, he formulated the main problem of differential calculus in the eighteenth and nineteenth centuries: «From the beginning of the 18th century upto the present time, the general task of differential calculus is usually formulated thus: how to find out the real equivalent for the

symbolic differential coefficient»⁴⁸. But although Marx also found real values of certain differential expressions, his attention was focused mainly on the relationship between algebraic and differential methods, and his presentation was subordinated to the clarification of their interaction in the field of differential calculus.

Marx treated symbolic differential expressions as *operational symbols*, i.e. they are «the symbols of processes, which are to be carried out»⁴⁹. The way of performing these operations was assumed, according to Marx, to be known to the addressee of his notes. The notion of the operational symbol was the basis for the notion of the operational formula and the operational equation. In the second sketch to the note «On the Differential», Marx not only discussed functions in general, but also considered a special kind of functions, namely, the formula of the sub tangent of the usual parabola.

The fact that the applicability of algebraic methods in differential calculus is not universal was shown by the fact that it is possible that quite correct algebraic transformations of a differentiable function result in the expression $\frac{0}{0}$, which is not mathematically correct. To avoid this, it is necessary to recognize that in the case when $\frac{\Delta y}{\Delta x} = \frac{0}{0}$, where the numerator and denominator are inseparable, it is necessary to switch to the use of differential methods, introducing the appropriate symbolism and terminology. Expressions of the specified type are renamed to $\frac{dy}{dx}$.

⁴⁸ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 31.

⁴⁹ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 31.

These expressions symbolized the transition to the field of differential calculus and indicated a certain «*operational strategy*» (*strategy of action*) for the preformation of differential expressions. Marx emphasized that «it becomes the starting point of differential calculus, now moving upon its own ground»⁵⁰. In this case, «the algebraic method, by itself, turns into its opposite, the differential method»⁵¹. Marx not only stated the transition of the algebraic method into the differential method, but also pointed to their interaction within the framework of differential calculus.

In the rough drafts of «On the Differential» one can find one of the first mentions of the notion of *inversion of the method (method reversal)*⁵². As inversion of the method Marx called the shift of the centre of gravity from algebraic transformations to differential transformations. Marx came to the definition of this philosophical notion in the process of expounding algebraic differentiation. In Marx's opinion, mathematicians had long been using this method without questioning its essence or formulating it explicitly. Marx substantiated the objective nature of this methodological technique and, moreover, the fact that the method of algebraic differentiation

⁵⁰ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 35.

⁵¹ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 31.

⁵² Amsterdam International Institute of Social History (IISG). A 104. See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 31. The notion of inversion of method, to which Marx, and after him Engels, attached fundamental importance in the whole complex of Marx's mathematical research, appeared not only in the rough sketches for the work «On the Differential», but also when commenting on the excerpted sources. As early as in the outlines of Sauri and Boucharlat's textbooks, Marx raised the question of a methodological technique that would allow one to move from algebraic to differential methods. The relation between differential and algebraic methods was formulated in a general form, which was the precondition for the formation of a generalized methodological notion, such as the notion of inversion of method.

he was developing synthesized historical methods developed in differential calculus.

Marx explained its essence in more detail. In particular, he wrote about the validity of the inversion of the method from a mathematical, or more precisely, from an algebraic point of view. «Since this inversion of method emerged from the algebraic movement of the function uz , its basis is thereby algebraic»⁵³.

Analyzing the methodological foundations of differential calculus, he remained first of all a philosopher-dialectician, which was fully manifested in his interpretation of the notion of inversion of method.

One could continue his reasoning about the inversion of method. Since the differential, as it has already been said, is the final point of algebraic development and also the starting point of differential calculus moving on its own basis, the inclusion of algebraic methods in differential calculus is connected with their subordination within the framework of differential calculus to differential methods. An analogy from the field of political economy comes to mind. In developing his system, Marx wrote that the relations of previous epochs, functioning within the framework of the bourgeois society already established, moving on its own basis, are reproduced as subordinate moments of bourgeois relations proper.

Thus, we have come very close to clarifying the status of the concept of inversion of method in the methodology of scientific cognition. The first thing to bear in mind is the deep connection of this concept with other methods developed by Marx.

⁵³ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 31, note.

Characteristically, the methodological techniques used by Marx are most often paired. Marx almost always took advantage of a twofold kind of analysis: logical research was complemented by historical research, the movement of theoretical thought from abstract categories to more concrete ones was interwoven with the opposite movement – from the concrete to the abstract. Marx insisted on the inadmissibility of absolutizing any one method of research, to some extent explained Marx's adherence to the use of dual methods in his analysis.

The notion of inversion of method was a generalization not only and not so much of mathematical methods, but first of all the result of reflections on the essence of dialectical methods of cognition. Therefore, the new methodological concept formulated in the 1880s should not be considered in isolation from other concepts of Marx's methodology, but, on the contrary, as one of its important moments. This statement is based on the modern interpretation of the inversion of the method⁵⁴.

The second outline of the manuscript «On the Differential» guessed the structure of Marx's final version of this note. Marx intended to specifically compare algebraic and differential methods on concrete examples of Newton-Leibnitz and Lagrange calculus. Thus, already from the analysis of the second draft of the manuscript «On the Differential» it follows that the basic concept of the historical sketch of the development of differential calculus grew out

⁵⁴ In particular, it is considered that the internal structure of the ascent from the abstract to the concrete is subordinated to the law of the inversion of method. In addition, it is emphasized that the inversion of method is associated with the transition to a qualitatively new level of cognition, the comprehension of old theories within the framework of new ones, and that it is the inversion of method that justifies the introduction of ideal objects into theory.

of Marx's positive presentation of his interpretation of the differential method. A parallel is suggested with his study of political economy, where the historical sketch was organically woven into the positive exposition of the theory. It is now clear that the same principle underlies the approach to this problem in the mathematical manuscripts. An analysis of the manuscript shows that Marx did not plan to stop at the results achieved, but intended to continue the research. Marx has here formulated plans for the structure of his current and future research ⁵⁵. Finishing the note «On the differential», Marx reported to Engels: «N. III The end of this second installment will follow after I look over John Landen in the Museum»⁵⁶. Researchers are inclined to believe that, since Marx was not able to see this book, the third section of the manuscripts was not written. It is not specified how Marx saw the continuation of his mathematical notes.

8. The addressee of Marx's mathematical notes and the addition to the note «On the Differential»

The crucial condition for the successful development of a scientific concept is the existence of a scientific community within which the results of scientific cognition can be tested. It is therefore understandable that Marx, who did not intend to publish his mathematical manuscripts, resorted to letters about mathematics as a

⁵⁵ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 61–62.

⁵⁶ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 39.

form of objectification of his views. The scientific team for Marx was Engels and their mutual friend and collaborator Samuel Moore.

In this connection, let us discuss the problem of the addressee of these manuscripts. It would seem that since Marx himself designated it by writing «For Fred» or «For General», there is no problem. But the Swedish researcher Sven-Erik Lidman in his work about Marx's and Engels's studies of natural science expressed a somewhat different point of view. He wrote: «Samuel Moore was a lawyer in Manchester, as well as a member of the First International Moore's authority was predominantly in mathematics. Marx paid more attention to mathematics than Engels and therefore shared his thoughts with Moore. Thus, he discussed with him the possibility of «determine mathematically the principal laws governing crises», one of Marx's most important problems in his mathematical pursuits. Moore was sceptical, and Marx abandoned the problem, albeit temporarily. Many years later, after Marx had developed his extensive mathematical manuscripts, he and Moore exchanged views and even writings on issues primarily related to differential calculus. However, Engels now played mainly the role of an intermediary»⁵⁷.

In evaluating this statement, one should distinguish between the facts and the hypotheses expressed. First, indeed, Marx and Engels recognized Moore as an authority in the field of mathematics, and Marx discussed with him the problem of mathematical derivation of the laws of crises, as evidenced by the above-mentioned letter of

⁵⁷ *Liedman S. E. E. Spiel der Gegensaetze. Frankfurt–New York. 1986. S. 125. Cf: Antonowa I.K. Wer war Adressat der mathematischen Lieferungen von Marx? // Naturwissenschaften und Produktivkraefte bei Marx und Engels. IMSF. Frankfurt am Main, 1991. S. 169–173.*

May 31, 1873.⁵⁸ Second, it is a fact that Engels, having received and studied Marx's mathematical manuscripts, forwarded them for review and evaluation to Moore (as after Marx's death he showed them to A. Sorge). And Moore, in turn, made a number of critical remarks both directly in the manuscript and separately. However, there is no reason to claim that Marx addressed these manuscripts to Moore, giving Engels the role of an intermediary. Lidman's hypothesis does not seem sufficiently convincing. There are both formal and substantive reasons for this. Thus, on purely formal grounds – by the dedications of Marx, we can conclude that they have a direct addressee – Engels, and we must also recognize that to communicate with Moore, Marx did not need to resort to Engels. Speaking about the substantive points, it should be taken into account that: the assessment of Engels' own role as a mere intermediary cannot be considered adequate. Incidentally, he, taking a direct part in this discussion on the method of differential calculus, not only managed to evaluate in his own way what Marx had done in this field, but also tried to penetrate into the essence of a new interpretation of the differential method. This is illustrated by the following letter from Engels to Marx on November, 21, 1882: «His [Moors] conclusion, namely that the algebraic method is only the differential method disguised, relates, of course, solely to his own method of geometrical construction and to that extent is fairly correct. I have written to tell him that you attach no importance whatever to the way in which one symbolises things in a geometrical construction, the application of the curves to the equations being, after all, sufficient. What's more,

⁵⁸ *Marx K., Engels F. Collected Works. LW 2010. Vol. 44. P. 504.*

I went on, the basic difference between your method and the old one consists in your transforming x into x' , i. e., making it a *true variable*, whereas the others proceed from $x + h$ which simply represents the sum of two quantities but never the variation of a quantity. Which is why your x , even though it has passed through x' and reverted to the first x , is nevertheless different from what it was before, whereas if one simply adds h to x and then subtracts it again, x remains constant throughout. But then any graphic depiction of variations must necessarily be the depiction of a *past* process, of the result, i.e. of a quantity that has become constant; the line x , the portion added to that constant, is depicted as $x + h$, two portions of one line. From this it clearly follows that a graphic depiction of how x becomes x' and then x again is impossible»⁵⁹.

Of great importance for studying the history of the creation of Marx's notes on mathematics is Moore's review of the mathematical manuscript «On the Differential». S. Moore made his comments on pages 12–13. There are pencil notes on the manuscript, also apparently attributed to Moore. In particular, he wrote about the need to give proof for the general form of the formulas⁶⁰. Marx's reply on the same day was resentful of Moore: «Sam, as you, too, immediately saw, criticises the analytic method as applied by me in as much as he quietly sets it aside and instead turns his attention to

⁵⁹ Marx K., Engels F. Collected Works. LW 2010. Vol. 46. P. 378–379.

⁶⁰ Engels forwarded all this material to Marx on November 21, 1882. In the letter Engels concerned himself with the graphical representation. If we look at Moore's own letters, it is clear that he was citing the same tangent problem with which Marx began his mathematical correspondence with Engels as early as 1863. This elementary problem had long been known to Marx.

the geometrical application, about which I said nothing at all»⁶¹. Nevertheless, Moore's judgement was very valuable. They caused some changes in the structure of Marx's mathematical research, leading to the appearance of «additions» to the note on the differential.

Meanwhile, four versions of Marx's additions to the notes on the differential are known, two of which are draft and two of which are fair-copy⁶². The question about the reasons and time of their writing is legitimate. There is reason to suppose that it was Moore's criticism that was the stimulus that made Marx once again turn to the problem of differentiation. In both the first and the second outlines Marx explicitly relied on the presentation in the manuscript «On the Differential», explaining the essence of the problems developed there: «In the last manuscript... it was essential for me to show ...that the algebraic method here applied itself turns into the differential method, owing to the fact that it develops inside the derivative, i.e., on the right hand side [we have] the *symbolic differential co-efficients* without the corresponding equivalent real co-efficients; along with this, these symbols, as such, become *independent starting*

⁶¹ Marx K., Engels F. Collected Works. LW 2010. Vol. 46. P. 380. To this we can only add that in the rough sketches on the differential the geometrical interpretation was also discussed, but in the white version Marx omitted this question.

⁶² Marx K. Mathematical Manuscripts. Calcutta. 1994. P. 62–63. The structure of the manuscript of «Addenda» changed in the process of work on it. In the first draft of the «Addenda» Marx explained the reason for not considering the geometric interpretation: «However, we shall keep it for the subsequent geometric application, since I personally wish to make a few more prefatory general remarks on the method used by me» (Amsterdam International Institute of Social History (IISG). A 104. Marx K. Mathematical Manuscripts. Calcutta. 1994. P. 254.)

points, and are given in the ready-made form of *operational formulae*»⁶³.

Most likely, in responding to Moore's criticism, Marx was addressing the question of the geometric interpretation of the operational formulas he was introducing: «However, I could have also indicated the geometric application of these operational formulae, since, for example, $y\frac{dx}{dy}$ serves as the *general formula for subtangents to curves*, which is, in form, entirely identical with $z\frac{du}{dx}$, $u\frac{dz}{dx}$, since all of them are products of a variable and a symbolic differential co-efficient»⁶⁴.

Thus, in order to understand the reasons for Marx's address to this problem in general, and, in particular, to the clarification of the essence of the inversion of method, the possibilities of geometric interpretation of the operational formulas, it is enough to recall that these very questions became Moore's arguments in his very restrained evaluation of the manuscript «On the Differential».

As regards the third and fourth drafts of the «additions», the first thing that strikes one is the character of fair-copy of both manuscripts. They are written with virtually no corrections, and if there were any, they were done in such a way that the original version was not visible. In addition, both versions are virtually identical in content. They are also identical in structure, which is basically different from that of the first and second drafts.

⁶³ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 62.

⁶⁴ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 62.

In these manuscripts, Marx omitted some of the differentiation problems discussed in the drafts. The word «additions» was missing from the headings. In both versions, Marx did not give a more general discussion of method as the essence of his discoveries in the manuscript «On the Differential». The question arises as to what motivated Marx to make two virtually identical copies of the manuscript. The absence of Marx's direct statements on this matter prevents us from giving a definitive answer. But a more or less plausible assumption can be made, first, by analyzing the differences in the two versions, and, in addition, the quality of the paper and the way the versions themselves were written.

In one of the drafts, designated by the researchers as the third, part of the text is written in English and part in German, while in the fourth the narrative was exclusively in German. In the third manuscript, the English text had the form of an address to a specific person, while in the fourth set there was no such address. In the fourth draft, however, the presentation was more complete: the names of those curves that could be described by functions that could be differentiated in a new way were given. Both versions are folded in half as if to be sent in an envelope. It is reasonable to conclude that Marx made two copies because from now on, after writing the second note «On the Differential», his correspondents became two individuals, Engels and Moore. It was to Moore, who spoke German, that Marx could address in his native English in those places that he considered most important.

If we accept this hypothesis, we must conclude that the third and fourth versions are equal versions of the Addenda, addressed to different persons. In addition, it would be possible to date these manuscripts more precisely, namely after November 22, 1882.

9. The central problem of the mathematical manuscripts

Taylor's theorem and its special case, MacLaurin's theorem, were central to Marx's mathematical manuscripts. Many pages of excerpts of both first and second order mathematical literature were devoted to their proof⁶⁵. Let us recall the main points from the history of Taylor's theorem. It deals with the decomposition of a function into a power series. The theorem was formulated by Taylor in his main work «Methodus Incrementorum Directa & Inversa» in 1715. Taylor himself did not appreciate the significance of the theorem he proved. It was named after him by N. Condorcet (1743–1794) in 1784, and in 1786 S. L'Huilier (1750–1840) called the power series obtained by decomposing a function «Taylor's series».

It is characteristic that Taylor himself formulated this theorem in the framework of finite difference calculus. It should be emphasized that finite difference calculus studies functions with discretely varying argument. This calculus emerged as an independent discipline in the early 18th century and was described by Newton. An analogue of the calculus of finite differences can be found in differential calculus because the algorithm for calculating finite differences is similar to the algorithm for differentiation.

⁶⁵ Marx drew on Th.G. Hall's textbook to investigate the problem, in particular by discussing the hypothetical character of the theorem and the transition from it to the binomial theorem and McLaurin's theorem. Amsterdam International Institute of Social History (IISG). A 104. See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 123–160, 214–236.

However, unlike differential calculus, there was no need for a special philosophical justification of this calculation, since the concept of infinitesimal value was not introduced.

One of the motivations for Marx's attention to Taylor's theorem was the fact that the theorem in some way approaches differential calculus from an algebraic perspective, and that differential and algebraic methods are intertwined. However, Taylor failed to substantiate the connection between differential and algebraic methods, and it remained at the level of hypothesis: «Taylor communicated it through a hypothesis, outwardly obtained with the help of a general proof»⁶⁶.

Assessing Lagrange's contribution to the proof of the theorem, Marx remarked: «He provides an *algebraic basis* to the differential calculus; but it is to be used only as the starting point, since it is quite pointlessly tedious to develop algebraically, that which can be attained much more easily, through the proper methods of differential calculus»⁶⁷. As for the connection between differential calculus and algebra, this connection seemed to them to be a given. Taylor and MacLaurin did not set themselves the task of justifying the connection of their conclusions with algebra and operated from the very beginning on the basis of differential calculus. The question of the necessity of algebraic verification of differential calculus belonged to Lagrange and, according to Marx's assumption, was preceded in this by John Landen, about the necessity of studying the work of whom Marx repeated in his manuscripts.

⁶⁶ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 224.

⁶⁷ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 218.

Lagrange introduced the concept of the derivative of a function. And Marx praised this contribution of Lagrange to differential calculus, stating that «the great merit of Lagrange lies not only in laying the foundations of Taylor's theorem and differential calculus in general through a purely algebraic analysis»⁶⁸. This was the mathematical aspect of the discovery. But there was also a philosophical aspect to it, namely that Lagrange's work objectively helped to answer Berkeley's philosophical objections to differential calculus. Taylor's theorem served for Marx as a basis for other philosophical generalizations. Thus, he wrote: «The real, and accordingly the simplest, interconnection between the new and the old, is always discovered only when this new itself already attains its final form, and it may be said, that in the differential calculus this return (taking) backwards was carried out by the theorems of Taylor and MacLaurin»⁶⁹.

Marx, like other researchers, was concerned with the question of whether Newton himself knew Taylor's theorem. Marx answered this question in the negative, giving a psychological justification for his belief: «To this it may be said with full confidence: that no, he was not one of those, who would give his pupils the opportunity to appropriate such a discovery. In reality he was still too absorbed with the elaboration of those very differential operations, which were already assumed to be well known and available to Taylor and MacLaurin. This is testified by the first elementary formulae of his calculus. Clearly, Newton initially approached them from the points

⁶⁸ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 91.

⁶⁹ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 90.

of departure of mechanics, and not from those belonging to pure analysis»⁷⁰.

However, the research of our time has shown that Marx was wrong. Thomas Whiteside (1932–2008) has discovered that in the XIV appendix of the unpublished version of «On the Quadrature of Curves» Newton in 1691–1692 solved a similar problem. Thus the notion of a «Taylor series» has little historical justification⁷¹. This circumstance in no way diminishes the importance of the discovery made by Taylor, who independently came to his results, as he informed his teacher John Machin (1686–1751) in a letter dated July 26, 1712. Lagrange not only generalized Taylor's theorem, but also found out the cases when it does not work. And in a number of manuscripts it was just about those cases when Taylor's theorem is inapplicable⁷². In this case, too, Marx was concerned with the question of the relation between algebraic and differential methods. He showed that Taylor's theorem is the most general and fruitful operational formula of the whole differential calculus.

Let us focus on one of the manuscripts on Taylor's theorem where Marx considered its generalizing role in differential calculus⁷³. The manuscript had two versions. One of them can be regarded as a draft and the other as a final version. Although the sketches were not dated by Marx himself, it is possible to establish the sequence of

⁷⁰ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 90.

⁷¹ See: *History of Mathematics from the Most Ancient Times to the Beginning of the 19th Century*. In 3 vol. - Moscow. 1970. Vol. 2: *Mathematics of the XVII century*. P. 295.

⁷² Amsterdam International Institute of Social History (IISG). A 104. See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 264–301, 237–238.

⁷³ Amsterdam International Institute of Social History (IISG). A 104. See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 237–238.

their writing on the basis of certain features of the manuscripts. One of the outlines had a number of features characteristic of a draft. Its reasoning did not constitute a complete statement of the problem. The form and appearance of the second version indicate that Marx apparently intended to present the problem in the form of notes-letters (Lieferungen). In this outline there was an indication of the second problem to be considered. This was, of course, MacLaurin's theorem, as indicated by one of the headings. Thus, this manuscript was, in our opinion, on a par with the manuscripts «On the Concept of the Derived Function» and «On the Differential» and its additions.

In this manuscript Marx did not only and not so much touch upon the problems of the proof of Taylor's theorem, but also clarified its connection with the binomial theorem. The importance of this manuscript was, among other things, that it addressed the general question of the concept of function and the peculiarities of the original function on which the differentiation operation is performed.

Thus, the interest in the central concept of the theory and the historical forms of its treatment was similar to Marx's studies in political economy, where the central concept was surplus value. Similar to his studies in political economy was Marx's turn to the history of differential calculus, which was another prerequisite for the formulation of his own understanding of the basic concepts of calculus.

10. Marx's manuscripts on the history of mathematics

Marx's engagement with the history of the theoretical discipline in which he was interested – be it political economy or mathematics – indicated not only and not so much a desire for

historical research as such, but also that the study of theoretical problems, in particular differential calculus, had progressed to a degree that both allowed the problems to be considered in retrospect and indicated the prospects for this development itself. This meant that Marx had reached a stage in his studies at which he had formed his own vision of the problems, including his own conceptual apparatus.

Similar to his research in political economy was Marx's study of the history of differential calculus, which was another prerequisite for formulating his own understanding of the basic concepts of calculus. From the point of view of his concept he evaluated the historical schools in the development of differential calculus. Focusing on the stages of its development, Marx did not concentrate on the conceptual apparatus of his predecessors or on the details of the development of differential calculus, but on the specifics of the method of differentiation used earlier, on the relation to Taylor's theorem.

Marx devoted an undated manuscript to the history of mathematics, which was a continuation of his reflection on the theory and history of differential calculus and in this sense had a complex character, reflecting all levels of his analysis of mathematics: the manuscript began with an outline and continued with his own reasoning, which became a prototype of a new treatment of the problem of differentiation. The presentation of his vision of the theory was interrupted in several places for excursions into the history of differential calculus⁷⁴.

⁷⁴ Amsterdam International Institute of Social History (IISG). A 104. See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 64–86.

Let us dwell briefly on the characterization of the structural parts of this work. Its excerpt part dealt with the problems of differentiation of trigonometric functions, which was a direct continuation of Marx's extracts in an earlier manuscript⁷⁵.

It was in this manuscript that Marx actively analyzed the essence of differentiation, namely, first of all, the conditions for replacing $\frac{0}{0}$ by $\frac{dy}{dx}$ and the meaning of both expressions. Marx was concerned with the question of at what stage we can say that there was a transition from algebra to differential calculus proper. Thus, this manuscript contained the first outline of a work on the differential and served as the beginning of a whole series of sketches on his own understanding of this basic concept of differential calculus.

The historical part of the presentation was based on the study of the subject in the treatises of Sauri and Boucharlat. To the manuscript, Marx enclosed a list of mathematicians who had played a significant role in the history of differential calculus. These included Newton, Leibnitz, Taylor, MacLaurin, Landen, D'Alembert, Euler, Lagrange, Poisson, Laplace, and F. Moigno⁷⁶.

In the manuscript itself, Marx elaborated on only a few of these names. And the first of them was Newton, in whose theory, as already mentioned, the special terminology of differential calculus

⁷⁵ See: *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 239–241. (fourth section). This was also evidenced by Marx's direct reference to this manuscript (thus, starting the manuscript, he writes «IV (continuation of IV A)»).

⁷⁶ F. Moigno (1804 – 1884) – a French Catholic priest, physicist and author. He considered himself a student of Cauchy.

was introduced for the first time – the concepts of fluxion and fluent, as well as the symbolism for differentials.

In analysing Newton's views, Marx not only described his views from his own or contemporary positions, but he himself took his point of view and considered in this terminology the differentiation of the product $y = uz$. Marx used the same example to demonstrate Leibnitz's concept.

Marx then highlighted the views of both mathematicians using the terminology that played a major role in his own research. Having begun his presentation of Newton's and Leibnitz's views, Marx did not develop it further, but moved on to formulate his own views in order to return to historical analysis. Without going into the differences between Newton's and Leibnitz's methods and comparing them as a whole with Lagrange's method, Marx introduced the term «preliminary derivative», which became one of the central terms in his treatment of the relevant problems.

Marx evaluated the history of the development of differential calculus in the same way as he evaluated the evolution of previous economic theories from the standpoint of the economic doctrine he had created at that time. In the history of differential calculus since the second half of the 17th century he called three stages – mystical, rational and purely algebraic. The basis of this periodization was the concept of differential and its various interpretations.

The first stage – mystical differential calculus – is associated with the names of its founders Newton and Leibnitz. They identified the differentials dx and dy with the increments Δx and Δy and interpreted them as infinitesimal quantities that are both equal to zero and different from it. The introduction of such quantities into differential calculus was not strictly mathematically justified, but was

accompanied by abstract reasoning, which, as mentioned above, allowed philosophers of the time, in particular Berkeley, to see differential analysis as a kind of idealistic system whose basic principles were not provable, but were accepted by mathematicians on faith, akin to the tenets of religion.

Marx wrote that Newton and Leibnitz «themselves believed in the mysterious character of the newly discovered calculus, which provided correct (and more over in the geometrical applications, really astonishing) results by a positively in correct mathematical procedure, They were thus self-mystified, valued the new discovery all the higher, enraged the crowd of old orthodox mathematicians all the more, and thus called forth the cry of oppsition; it aroused an echo even in the lay world, and that is necessary for paving the path for something new»⁷⁷. At the same time, according to Marx, «while the inventors of differential calculus and the majority of their followers make differential symbols the point of departure of calculus, Lagrange conversely takes algebraic deduction of the real functions of the independent variables as his starting point, and makes the differential symbols purely symbolic expressions of the derived functions»⁷⁸.

The second stage is rational differential calculus, associated mainly with the works of D'Alembert, who was based on Newton and Leibnitz. Marx wrote: «D'Alambert starts directly from the *starting point of Newton and Leibnitz*»⁷⁹.

⁷⁷ Marx K. Mathematical Manuscripts. Calcutta. 1994. P. 78.

⁷⁸ Marx K. Mathematical Manuscripts. Calcutta. 1994. P. 50.

⁷⁹ Marx K. Mathematical Manuscripts. Calcutta. 1994. P. 79.

The third stage – purely algebraic differential calculus – owes its origin primarily to Lagrange, who tried to reduce differential calculus «to a strictly algebraic foundation»⁸⁰. Assessing Lagrange's achievements, Marx wrote that «Lagrange is in fact free of all that appears to him as metaphysical transcendence in Newton's fluxions, Leibnitz's infinitesimals of various order, the theory of limits of vanishing magnitudes, the substitution of the symbol $\frac{0}{0}$ ($= \frac{dy}{dx}$) in place of the differential co-efficients etc. However, thereby he himself is not deterred from constantly using one or the other of these «metaphysical» notions, while he applies his theory to curves etc»⁸¹. In addition, the unsatisfactoriness of Lagrange's theoretical constructions consisted, according to Marx, in the insufficient justification of the theory of series.

Characterising the general course of the development of differential calculus, Marx drew a parallel with the history of German classical philosophy: «Thus did Fichte side with Kant, Schelling – with Fichte and, Hegel with – Schelling, wherein neither Fichte, nor Schelling, nor Hegel did investigate the general basis of Kant, of idealism in general; or else they could not have developed in further»⁸².

* * *

⁸⁰ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 90.

⁸¹ Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 92.

⁸² Amsterdam International Institute of Social History (IISG). A 104. *Marx K. Mathematical Manuscripts*. Calcutta. 1994. P. 94.

The main subject of Marx's works on mathematics was not even the mathematical concepts themselves, but rather the method, the process of differentiation of functions and, in this connection, a new method of differentiation, which took into account the peculiarities of the relationship between algebraic and differential methods. Marx established the boundary between them and gave a proof of the fact that the relationship between these methods is much more complex than mathematicians understood it, and that in differential calculus there is an inversion of method. It is characteristic that Marx not only stated the transition of the algebraic method into the differential method, but also pointed to their interaction within the framework of differential calculus. Again, while analyzing the methodological foundations of differential calculus, he remained a dialectical philosopher, which was fully manifested in his interpretation of the notion of inversion of method. It is safe to say that the notion of inversion of method was a generalization not only and not so much of mathematical methods, but above all the result of thinking about the essence of dialectical methods of cognition in general.

11. History of the publication of K. Marx's «Mathematical Manuscripts»

In the history of the publication of K. Marx's Mathematical Manuscripts, science, ideology and politics were intertwined as never before. References to the authority of scientists who did not appreciate the significance of the manuscripts and delayed their publication, attempts to point to new names, attributing to them the glory of primacy in preparing for the publication of this most problematic part of Marx's theoretical heritage, an inaccurate and

incomplete misleading presentation of the facts of the history of the study and publication of Marx's mathematical manuscripts, – all this is only an echo of the events that the world has experienced over the past century. It is precisely these circumstances that make the task of documenting the process of preparing Marx's manuscripts for publication extremely urgent.

Although the main subject of K. Marx's research since the 1850s was the foundations of political economy, he did not exclude the possibility of publishing the results he had achieved in a field other than political economy. After his death, Engels, referring to Marx's youngest daughter Eleanor Eveling, wrote to his middle daughter, Laura Lafargue on June 24, 1883: «After poor Mohr's death, on my inquiry, Tussy informed me that he had told her, she and I were to take possession of all his papers, and procure the publication of what was to be published, especially the 2nd volume [of Capital] and the mathematical works»⁸³.

Nevertheless, up to now the mathematical manuscripts of K. Marx have not been published in full either as a separate book or as part of any of the collected works of K. Marx and F. Engels. And finally, for several decades now the mathematical manuscripts of K. Marx have been awaiting their publication as part of the Complete Works of K. Marx and F. Engels in the original languages (MEGA²)⁸⁴.

⁸³ *Marx K., Engels F. Collected Works. LW 2010. Vol. 47 P. 39.*

⁸⁴ In the USSR, fragments of manuscripts were published beginning in 1933 for the first time. See: Математические рукописи. Конец 1870–начало 1880 г. *Маркс К. Производная и символический дифференциальный коэффициент // Под знаменем марксизма, 1933. № 1. С. 15–73.* (Mathematical Manuscripts. Late 1870–early 1880. Marx K. The Derivative and the Symbolic Differential Coefficient. // Under the Banner of Marxism, 1933. № 1. P. 15–73). Обоснование

12. The first attempts to publish Marx's mathematical manuscripts. E.J. Gumbel's work on their systematization

Until the end of his life, F. Engels did not forget about the wish of his comrade to publish mathematical manuscripts and on September 23, 1885, in the preface to the second edition of «Anti-Dühring» he informed readers that he intended to publish «Dialectics of Nature» «perhaps [vielleicht] in conjunction with the extremely important mathematical manuscripts left by Marx»⁸⁵.

After the death of F. Engels, some of Marx's mathematical manuscripts ended up in the hands of E. Bernstein. As a result of collecting activities, a large number of documents by K. Marx and F. Engels were delivered to the archives of the Social Democratic Party of Germany (SPD) in July 1925, among which were 10 notebooks by K. Marx on mathematics. They were immediately photocopied for the Institute of K. Marx and F. Engels at the All-Russian Central Executive Committee of the RSFSR (IME), founded in Moscow in 1921.

дифференциального исчисления // «Марксизм и естествознание». М. 1933. С. 4–61. (Justification of Differential Calculus // Marxism and Natural Science. Moscow, 1933, P. 4–61). О понятии функции. [Отрывок из алгебраических тетрадей] // Вопросы философии. 1958. № 11 С. 92–95 (On the Concept of Function. [Excerpt from Algebraic Notebooks] // Questions of Philosophy. 1958. № 11, P. 92–95). Норма прибавочной стоимости и норма прибыли, трактуемые математически. [Отрывки из неопубликованной тетради К. Маркса] // Лопырев Н.К. Развитие теории производительности труда. М. 1965. (The Rate of Surplus Value and the Rate of Profit, Treated Mathematically. [Excerpts from an Unpublished Notebook by K. Marx] // Lopyrev N.K. The Development of the Theory of Labor Productivity. Moscow, 1965). К. Маркс. Математические рукописи. М. 1968. (K. Marx. Mathematical Manuscripts. Moscow. 1968).

⁸⁵ Marx K., Engels F. Collected Works. LW 2010. Vol. 25. P. 13.

It was previously said that in 1882 S. Moore expressed his assessment of Marx's manuscripts. Now to assess the significance of Marx's manuscripts on mathematics, the Board of the Social Democratic Party of Germany commissioned Martin Leo Arons (1860–1919), a physicist and member of the SPD, to express his opinion on them. He described them as the work of a student⁸⁶. No later than 1914, D.B. Ryazanov proposed to Friedrich Adler (1879–1960) to publish one short mathematical note by Marx that would not require extensive editorial processing⁸⁷. But this first publication did not take place. Apparently, it was precisely the lack of a positive assessment of the manuscripts that removed plans for their publication from the agenda for some time. But not even a few decades had passed before questions about the scientific significance of these works of Marx and their publication were raised again, this time in the USSR.

This was partly due to the idea to publish «all of Marx», formulated in the decision of the Comintern in July 1924. This project became one of the main tasks of the IME, which actively collaborated with the Institute for Social Research in Frankfurt am

⁸⁶ See: MEGA². I/26. S. 595–596. It is also known that A. Einstein agreed with this opinion. F. Mehring wrote: «Marx also sought intellectual recreation on quite a different field, namely mathematics. Particularly in times of mental anguish and other sufferings he would seek consolation in mathematics, which exercised a soothing effect on him. Engels and Lafargue both contend that he made independent discoveries on this field, but this is beside the point here, and mathematicians who went through his MSS. after his death are reported not to have endorsed this opinion» (Mehring F. Karl Marx: The Story of His Life. Covivi, Friede Publishers, 1935. Ch. 15.).

⁸⁷ См.: Гумбель Э. О математических рукописях К. Маркса (сообщение) // Летописи марксизма. (Записки Института К. Маркса и Ф. Энгельса). Кн. 3. М.–Л. 1927 г. С. 56. (Gumbel E. On the Mathematical Manuscripts of K. Marx (report) // Chronicles of Marxism. (Notes of the Institute of K. Marx and F. Engels). Book 3. M.–L. 1927. P. 56).

Main and the Berlin Archives of the Social Democratic Party in Germany⁸⁸.

It should be borne in mind that at that time there were in USSR several opportunities for publishing the works of K. Marx and F. Engels. In addition to the collected works, the Archive of K. Marx and F. Engels was published in Russian and German⁸⁹, and a number of theoretical magazines on the history and theory of Marxism were available, in particular, the magazines «Pod Znamenem Marxizma» (Under the Banner of Marxism) (1922–1944) and «Letopisi Marxizma» (Chronicles of Marxism) (1926–1930). Communists from different countries took part in the work on the publication of the works of K. Marx and F. Engels. High demands were placed on those wishing to work on this project, including not only an academic education and knowledge of foreign languages, but also the ability to put aside their own interpretations and expositions of the published works and to comprehend the views of K. Marx and

⁸⁸ See : *Hecker R.* Erfolgreiche Kooperation: Das Frankfurter Institut für Sozialforschung und das Moskauer Marx-Engels-Institut (1924-1928) // Beiträge zur Marx-Engels-Forschung. Neue Folge. Sonderband 2. Berlin 2000. S. 85–86. MEGA¹ published only 7 volumes (8 books) of the first section and four volumes of correspondence. In total, 11 volumes (12 books) were published at first in cooperation with Marx-Engels Archive Publishing Company, which had been founded in 1924 together with the Frankfurt Institute for Social Research. In 1928, both institutes dissolved their articles of association. It should not be forgotten that the SPD was banned in June 1933.

⁸⁹ The magazin «Chronicles of Marxism» gave the following description of the first volume of the German edition of the «Archive...»: «The first volume of the German «Marx-Engels Archiv» is the first appearance of the Institute of K. Marx and F. Engels on the international front of Marxism. This magazin should have the same significance for the West as the Russian «Archive» for Russian readers; it is primarily a historical magazin, in which articles and studies, documents and materials are published, concerning the origin, development and dissemination of the ideas of scientific socialism or, in other words, concerning the history of Marxism in theory and practice» (Chronicles of Marxism. Moscow-Leningrad, 1926. P. 137).

F. Engels themselves. Thus, the publication and study of the manuscript heritage of K. Marx and F. Engels, which began in the 1920s, gave a start to understanding the mathematical manuscripts and implementing plans for their publication.

The study, systematization of mathematical manuscripts and the preparation of their fragments for publication began almost immediately after the IME received photocopies in 1925⁹⁰. This work was entrusted to the German mathematician Emil Julius Gumbel (1891–1966). It should be said that E.J. Gumbel was both a scientist and university teacher, as well as a publicist, and generally a notable figure in the anti-fascist movement in Germany. His merits were recognized not only by friends, but also by enemies. In the Gestapo documents of 1933, which are at the disposal of Russian State Social and Political History Archives (RGASPI), E.J. Gumbel was given the following description:

*«Before 1933: Pacifist writer, author of numerous works inciting hatred, member of the revolutionary union of artists. Professor at the University of Heidelberg. After 1933: emigrated to France in 1933. Works at the University of Lyon, reads reports in Masonic lodges in France. Deprived of citizenship on August 23, 1933»*⁹¹.

In October 1925, E.J. Gumbel came to Moscow on his own initiative, using the opportunities of his six-month sabbatical from

⁹⁰ And now RGASPI only stores copies of the mathematical manuscripts. Their originals are currently in the Amsterdam International Institute of Social History (IISG) and are available on the website www.iisg.amsterdam.

⁹¹ *«Vor 1933: Pazifistischer Schriftsteller und Verfasser zahlreicher Hetzschriften, Mitglied des revolutionären Künstlerbundes. Prof. an der Universität Heidelberg. Nach 1933: Emigrierte 1933 nach Frankreich und ist an der Universität Lyon tätig, hält Vorträge in französischen Freimaurerlogen. Wurde am 23.8.33. ausgebürgert»* (RGASPI. F. 191. Inventory 2 (supplement). Case 435. Sh. 581).

Heidelberg University. Documents from inventory 50 of fund 71 of the RGASPI testify to E.J. Gumbel's collaboration with the Institute of K. Marx and F. Engels. Initially, the purpose of Gumbel's visit to Moscow was to give lectures on biology, relativity theory, statistics and political issues at one of the USSR's universities, which he had asked the Soviet authorities about back in 1924. A. Einstein wrote a letter of recommendation to Gumbel. In 1925, Gumbel also offered his services in publishing the mathematical manuscripts of K. Marx. «In the end, I would be happy to publish the mathematical notes contained in K. Marx's legacy», he wrote⁹². There was no talk of «getting» Gumbel, who himself offered his services⁹³. And this work was entrusted to him.

The scientist was faced with the following tasks: «to arrange all the material in a systematic order» and «to prepare the relevant part for publication». In addition, D.B. Ryazanov expressed a desire to receive a detailed but laconic description of all the manuscripts, so that they would all be described already at the first, albeit partial, publication.

Systematizing the mathematical manuscripts of K. Marx on the whole, Gumbel arranged them in the following order: calculations and excerpts, rough drafts, independent mathematical works. He believed that only those manuscripts that did not contain

⁹² «Endlich wäre ich gerne bereit, die im Nachlass von Karl Marx vorhandenen mathematischen Notizen herauszugeben» (RGASPI. F. 71. Inventory 50. Case 148. Sh. 5).

⁹³ «Für die Herausgabe der mathematischen Manuskripte von Marx wurde 1925 der deutsche Mathematiker Emil Julius Gumbel gewonnen» (*Vogt A. Emil Julius Gumbel: der erste Herausgeber der mathematischen Manuskripte // MEGA-Studien. 1995 / 2. S. 27. (Vogt A. Emil Julius Gumbel: the first editor of the mathematical manuscripts // MEGA-Studien. 1995 / 2. S. 27.)*)

calculations or excerpts could be considered independent works of K. Marx. Gumbel did not consider it expedient to publish the remaining manuscripts other than these «independent» notes.

Not being a historian of mathematics or a philosopher, he nevertheless assessed the significance of Marx's mathematical manuscripts as follows: «Marx was mainly concerned with those works that were already known at the end of the 18th century. In choosing them, as is clearly evident, Hegel influenced him. Hegel names the same works by Euler, Lagrange, Landen, Newton and is also occupied with the foundations of differential calculus and the derivation of the derivative. Two new works, from 1848 and 1852 (Th.G. Hall and G.W. Hemming), which Marx read, are of no significance. Marx did not know the mathematical literature that immediately preceded or was contemporary with him, such as Gauss and Bolzano»⁹⁴.

E.J. Gumbel's stay in Moscow, as he had planned from the very beginning, was not long. From his correspondence with the deputy director of the IME Ernst Czóbel (1886–1953), it follows that already at the beginning of April 1926, i.e. six months after his arrival, Gumbel completed his work in Moscow and returned to Germany⁹⁵.

⁹⁴ Гумбель Э.Ю. О математических рукописях К. Маркса (сообщение) // Летописи марксизма. (Записки Института К. Маркса и Ф. Энгельса). Кн. 3. М.-Л. 1927 г. С. 59. (E.J. Gumbel, On the Mathematical Manuscripts of K. Marx (report) // Chronicals of Marxism. Moscow-Leningrad 1927. Book 3. P. 59).

⁹⁵ The work of E.J. Gumbel at the IME also resulted in 15 articles on the Soviet Union and the article «Marx as a Statistician» (Marx als Statistiker). In addition, he published an article on mathematics in the «Matematicheskii Sbornik». See: E.J. Gumbel Eine Beziehung zwischen Fehlermassen [A Relationship between Error Measures] // Matematicheskii Sbornik. Vol. 33. No. 4. Moscow, 1926. P. 395–412.

In Germany, he continued to study Marx's manuscript legacy in the SPD archive, as well as work on his article, which he promising ambitious entitled «The Mathematical Manuscripts of Marx and Engels. Published by E.J. Gumbel. Heidelberg» (Die mathematischen Manuskripte von Marx und Engels. Herausgegeben von E.J. Gumbel. Heidelberg). Of this manuscript, only the first page with the aforementioned title and the beginning of the introduction is available at RGASPI. But most importantly, right under the title was a list of the sections of the article revealing its actual content:

- I. List of manuscripts.
- II. II. Description of drafts.
- III. III. Reproduction of two drafts⁹⁶.

A year later, in 1927, in the midst of Gumbel's work on his extensive article, his report «On the Mathematical Manuscripts of K. Marx» was published in the «Chronicles of Marxism»⁹⁷. It was included as an integral part of the article that Gumbel was working on⁹⁸.

⁹⁶ Initially, E.J. Gumbel's article had the following structure: 1) Mathematical manuscripts of Marx and Engels (6 pages), 2) commentary 33 (pages), 3) two notes (2 pages), 4) manuscript No. 4 with Gumbel's notes (28 pages). The drafts that were supposed to be published later received the editorial titles «On the Differential» (manuscript No. 1, according to Gumbel's systematization) and «Differentiation of a Product» (Differentiation eines Produkte) (manuscript No. 4). Both drafts were written fair-copy by Marx. During the course of the work, the structure of Gumbel's article was repeatedly changed.

⁹⁷ *Gumbel E.J.* On the Mathematical Manuscripts of K. Marx (report) // *Chronicles of Marxism*. (Notes of the Institute of K. Marx and F. Engels). Book 3. Moscow-Leningrad. 1927. P. 56–60. The author's copy of the journal was sent to E.J. Gumbel on May 25, 1927 (RGASPI. Fund 71. Inventory 50. Case 148. Sh. 100).

⁹⁸ In particular, a paragraph included in the article on the recommendation of D.B. Ryazanov was repeated: «Among Marx's manuscripts, located in the archive of the Social Democratic Party of Germany, D.B. Ryazanov found a large number of notebooks and notes of mathematical content. Before the start of the war, he took steps to publish them, but he was not successful at the time. Thus, he

Initially, it was planned to publish K. Marx's mathematical manuscripts in Volume XVI of the Complete Works of K. Marx and F. Engels in the original languages (MEGA¹), but soon, as was known already in October 1926, this plan was abandoned, as was later the entire edition of MEGA. It should be understood that only Marx's works on mathematics could be published in the collected works of Marx and Engels.

As for extensive article, and in this case it was precisely an article by Gumbel that was being discussed – «Your work on Marx» («Ihre Arbeit über Marx» – RGASPI. F. 71. Inventory 50. Case 148. Sh. 57), there is no doubt that Marx's drafts (ready for publication from the very beginning) constituted a smaller part of it. This is also evidenced by Gumbel's letters, which discuss the proofreading he undertook in the layout of his article. Although major revisions to the article were not intended⁹⁹, Gumbel reported on his addition of new pages, notes and sections to the article, the rearrangement of some paragraphs, etc., which would have been impossible if it had been a question of publishing Marx's manuscripts¹⁰⁰.

suggested to F. Adler to print one mathematical work of Marx, fully processed for publication.” (*E.J. Gumbel. On the Mathematical Manuscripts of K. Marx* (report) // *Chronicals of Marxism. Moscow-Leningrad 1927. Book 3. P. 56*). See also RGASPI. F. 71. Inventory 50. Case 148. Sh. 32–33, 73. D.B. Ryazanov spoke with F. Adler about publishing a draft that was marked by Gumbel as «manuscript no. 4» and entitled «Differentiation of a Product» (Differentiation eines Produkte).

⁹⁹ «A cursory review shows us that some small, purely technical and editorial changes (bibliographical and similar) still need to be made» (Eine flüchtige Durchsicht zeigt uns, dass darin noch einige kleine rein technisch-redaktionelle Aenderungen (bibliographisches und ähnliches) vorzunehmen sind» (RGASPI. F. 71. Inventory 50. Case 148. Sh. 57).

¹⁰⁰ RGASPI. F. 71. Inventory 50. Case 148. Sh. 55, 67. E.J. Gumbel discussed the changes to the article with Ernst Czóbel. The following employees of the IME expressed their comments on Gumbel's article: philosopher, translator, and journalist A.M. Woden (1870–1939), who worked at the institute until 1931 and took part in preparing for publication volume I/1.1 of MEGA¹, where K. Marx's

Due to the author's constant additions and corrections during the proofreading process, the publication was moved from volume 2 to volume 3 of the German edition of the «Archive of K. Marx and F. Engels». As a result of critical comments on these changes from the IME, it was decided to publish it in its original version. So in one of the letters sent to E.J. Gumbel from the IME, it was reported: «Due to additional insertions and rearrangements, your article was substantially changed both in the typographical and editorial sense, so that you forced us, especially Comrade Ryazanov, to consider whether this work could be accepted into the Archive without a second oral discussion about its revision. Ryazanov approved it in its previous form. In its new form it seemed to him completely alien, difficult to survey, distorted... The revision claims to have given a historical and theoretical appreciation of Marx's mathematical works, which is actually not the case... The idea also arose that comrade Ryazanov would write a historical essay on Marx's studies in mathematics based on numerous facts known to him, and your participation, purely textual and not publishing, could be useful»¹⁰¹.

doctoral dissertation was published, and the German Marxist A. Thalheimer (1884–1948).

¹⁰¹ «Sie haben durch Ihre nachträglichen Einfügungen und Umstellungen Ihre Arbeit so wesentlich umgewälzt, sowohl in typographischer wie in redaktionellen Hinsicht, dass Sie uns, speziell den Gen. Rjazanov dazu zwangen, die Frage in Erwägung zu ziehen, ob die Arbeit ohne neuerliche mündliche Besprechungen über Umarbeitung in das Archiv aufgenommen werden kann. In der alten Form hat Rjazanov sie genehmigt, in der neuen Form erschiehn sie ihm ganz fremd, unübersichtlich, verzerrt... Die Umarbeitung macht Anspruch auf den Anschein, eine historische und theoretische Würdigung der mathematischen Arbeiten von Marx gegeben zu haben, was tatsächlich nicht der Fall ist... Es tauchte auch der Gedanke auf, dass Genosse Rjazanov eine historische Skizze über Marxens Beschäftigung mit Mathematik auf, auf dem Grund ihm bekannten zahlreicheren Angaben macht, und Ihre Mitarbeit lediglich als eine textkritische, und nicht als eine herausgeberische nutzbar gemacht werden soll» (RGASPI. F. 71. Inventory 50. Case 148. Sh. 106, 106rs.). Subsequently, Gumbel was informed of the dates of

Already from this letter of July 21, 1927 it becomes clear that D.B. Ryazanov was not satisfied with Gumbel's interpretation of mathematical manuscripts, and no longer envisaged Gumbel's participation in the work on the publication of Marx's mathematical manuscripts.

The publication of Gumbel's article was planned for the 4th volume of the edition. In the magazine «Chronicles of Marxism» Book 3 for 1927, an announcement was even published of the fourth book of the «Archive of K. Marx and F. Engels», where in the second section of this edition, among other documents, the following publications were planned: «From the literary legacy of Marx and Engels... Supplements to «Dialectics of Nature». K. Marx. Mathematical Manuscripts»¹⁰².

However, E.J. Gumbel's article never came out either in the USSR or in others countries, although it was already at the typesetting stage. Consequently, the two drafts of Marx that were planned to be included in it were not published either. The reason that E.J. Gumbel's article in the «Archive of K. Marx and F. Engels» was not published was that D.B. Ryazanov's consent for its publication was not obtained. The then head of K. Marx's research library at the IME, Soviet mathematician and philosopher E. Kolman (1892–1979), gave his assessment of this situation. He believed that D.B. Ryazanov delayed publishing the manuscripts because he was not sure of their scientific significance and was afraid of harming

Marx and Engels' letters, references to their works, where mathematical manuscripts were discussed, and he included them in the article.

¹⁰² Chronicles of Marxism. (Notes of the Institute of K. Marx and F. Engels). Book 3. Moscow-Leningrad: State Publishing House. 1927, P 1rs. The manuscripts were not intended to be published in full, but only in fragments.

Marx's authority. The conclusion suggests itself that E.J. Gumbel's article about K. Marx's studies in mathematics seemed to D.B. Ryazanov found it unconvincing because it did not prove the «scientific significance» of these manuscripts by K. Marx, which made the publication of disparate sketches devoid of political and ideological meaning. There was also a purely formal reason. After the merger of the IME and the Lenin Institute on November 3, 1931, the key features of the magazine «Archive of K. Marx and F. Engels» were changed and the article by the German mathematician no longer corresponded to them: «From now on, only manuscripts from the literary heritage of the founders of Marxism were published in it, and research articles, reviews, etc. were no longer published. At the same time, the publication of the magazine «Chronicles of Marxism» ceased»¹⁰³. E. J. Gumbel was also denied the right to continue working at the IME. One can only assume that the basis for this was precisely the formal approach¹⁰⁴ of the German mathematician to processing the manuscripts, which could not but affect the subsequent corresponding assessment by the scientific community of the significance of Marx's manuscripts.

From this it follows that E.J. Gumbel can rightfully be considered one of the first scientists who began work on the systematization of Marx's mathematical manuscripts and the preparation of a certain part of them for publication, while at the

¹⁰³ Литературное наследство К. Маркса и Ф. Энгельса. История публикации и изучения в СССР / Сост. Е. Кандель. М.: Политиздат. 1969. С. 160. (Literary Heritage of K. Marx and F. Engels. History of Publication and Study in the USSR / Comp. E. Kandel. Moscow. 1969. P. 160).

¹⁰⁴ E.J. Gumbel himself spoke about this: Летописи марксизма. (Записки Института К. Маркса и Ф. Энгельса). Кн. 3. М.-Л.1927 г. С. 56. (Chronicles of Marxism. (Notes of the Institute of K. Marx and F. Engels). Book 3. Moscow-Leningrad. 1927. P. 56).

same time it is only a big exaggeration that one can call him their first publisher¹⁰⁵.

The publication of Marx's mathematical works required an understanding of their internal logic and structure and was the result of the work of many specialists: textual scholars-decipherers and mathematicians who gave the manuscripts both a professional historical-mathematical and philosophical assessment.

13. The first publication of fragments of mathematical manuscripts by K. Marx in Russian by a team of mathematicians led by S.A. Yanovskaya

The Soviet and Russian mathematician N.Kh. Rozov (1938–2020) wrote in 1968: «It took decades of persistent, painstaking work by many people before we managed to sort out, systematize, decipher, and date about 1,000 sheets of photocopies of mathematical manuscripts, identify the sources that Marx studied and took notes on, separate Marx's own comments from numerous excerpts, prepare a thorough translation for publication, and compile a commentary.

¹⁰⁵ *Vogt A.* Emil Julius Gumbel: der erste Herausgeber der mathematischen Manuskripte // MEGA-Studien. 1995 / 2. S. 26–41. (*Vogt A.* Emil Julius Gumbel: the first editor of the mathematical manuscripts // MEGA-Studien. 1995 / 2. S. 26–41). In this article the author reports: «In June 1990, the author found documents in Moscow that clearly prove Gumbel's "authorship" in preparing the manuscripts for printing» (Die Autorin fand im Juni 1990 in Moskau Dokumente, die die "Urheberschaft" Gumbels bei der Vorbereitung der Manuskripte zum Druck eindeutig beweisen») (S. 27). An analysis of the above-mentioned documents showed that there was no talk of any preparation of Marx's mathematical manuscripts for publication.

Another significant difficulty was that Marx did not prepare any of his mathematical manuscripts specifically for publication»¹⁰⁶.

The beginning of the work of Soviet mathematicians on the publication of mathematical manuscripts can be dated back to 1931. The tragic story of the arrest of the director of the IME D.B. Ryazanov that year, also sheds some light on the fate of K. Marx's manuscripts on mathematics. Let us recall that the director of the IME, among other things, was accused of keeping unpublished letters of K. Marx in his personal safe. E. Kolman wrote in his memoirs that Ryazanov's hiding place «contained photocopies of several letters from Marx and Engels, as well as Marx's mathematical notebooks and his notes on natural science issues»¹⁰⁷. As was said above, D.B. Ryazanov himself wanted to write a work on K. Marx's studies in mathematics¹⁰⁸. There is reason to believe that the arrest of D.B. Ryazanov did not delay, but rather pushed the new leadership of the Institute to resume work on preparing mathematical manuscripts for publication¹⁰⁹. Their publication and especially the commentary from the standpoint of Marxist dialectics also responded to the tasks of the ideological struggle in mathematics that was waged in the USSR in the 1930s.

¹⁰⁶ *Розов Н.Х.* Математические рукописи Карла Маркса // Успехи математических наук (УМН). 1968.Т. 23. Выпуск 5. С. 205. (Rozov N.Kh. Mathematical manuscripts of Karl Marx // Uspekhi matematicheskikh nauk (UMN). 1968. Vol. 23. Issue 5. P. 205).

¹⁰⁷ *Кольман Э.* Мы не должны были так жить! М. 2011. С. 228. (Kolman E. Die verirrte Generation. So hätten wir nicht leben sollen !). The odious nature of this man does not, however, exclude the value of his information about the events of which he was a direct witness, without taking into account his often negative assessments of his contemporaries, in particular, the same E.J. Gumbel.

¹⁰⁸ RGASPI. F. 71. Inventory 50. Case 148. Sh. 106, 106rs.

¹⁰⁹ *Кольман Э.* Мы не должны были так жить! М. 2011. С. 229. (Kolman E. Die verirrte Generation. So hätten wir nicht leben sollen !).

E. Kolman stated that it was he who advised V.V. Adoratsky (1878–1945), who replaced D.B. Ryazanov and headed the Marx–Engels–Lenin Institute (IMEL) in the period 1931–1939, to create a team to decipher these manuscripts, as a result of which work on them was entrusted to a group of Soviet scientists from the Moscow State Lomonosov University (MSU). The work was led by mathematician and philosopher Sofia Aleksandrovna Yanovskaya (1896–1966); her team also included Dmitry Abramovich Raikov (1905–1980), a Soviet mathematician and specialist in functional analysis, probability theory, and general topology, and Anna Natanovna (Iovnasovna) Nakhimovskaya (1908–2005), a Soviet and Belarusian mathematician and geometer, and later an associate professor at the Faculty of Mechanics and Mathematics at Belarusian State University.

The first partial publication of Marx's mathematical manuscripts was in Russian, and it took place in 1933 in two editions: in the magazine «Pod Znamenem Marxizma» and in the collector «Marxism and Natural Science»¹¹⁰. The publication of the manuscripts was timed to coincide with the 50th anniversary of Marx's death. The introductory article stated: «Marx's mathematical works, which are independent in nature, fall into two parts. The manuscripts dating back to an earlier period deal with the lines of

¹¹⁰ Под знаменем марксизма. Философский и общественно-политический журнал. № 1. 1933. С. 15–73. (Under the banner of Marxism. Philosophical and socio-political journal. No. 1. 1933. P. 15–73). Марксизм и естествознание. М. 1933. С. 3–61. (Marxism and Natural Science. Moscow, 1933. P. 3–61). A facsimile of one of the pages of Marx's manuscript was given as illustration. The collector also published an article by S.A. Yanovskaya, «On the Mathematical Manuscripts of K. Marx».

Taylor and Maclaurin and Lagrange's Theory of Analytical Functions. In the later manuscripts, Marx gives his own justification of differential calculus. This is precisely what is discussed in the correspondence between Marx and Engels (letters of August 18, 1881 and November 21 and 22, 1882). This last part of the works is in turn divided into three parts, of which the third (historical essay) is only in rough draft. The first two are printed according to the text that Marx made a fair-copy for Engels. In the appendix to both the first and the second, there are versions explaining the text and individual notes from Marx's rough manuscripts. The third part is given according to the rough draft... Marx's mathematical manuscripts will be published in full in one of the next volumes of the «Marx–Engels Archives». Already in this publication one can see the structure of the manuscripts, which scholars will adhere to when preparing a later editions of the «Mathematical Manuscripts». The titles of Marx's works were compiled by the publishers: I) The Derivative and the Symbolic Differential Coefficient. Algebraic Differentiation of the Simplest Functions, II) The Differential and Differential Calculus. Inversion of the method. The Differential, III) Historical Essay.

In addition, in the footnotes, the publishers corrected inaccuracies, or as they were called in the publication, «typos» of Marx. Then followed the article by S.A. Yanovskaya «On the Mathematical Manuscripts of K. Marx»¹¹¹. It is worth citing the full words of S.A. Yanovskaya about how the work on the publication

¹¹¹ Под знаменем марксизма. Философский и общественно-политический журнал. № 1. 1933. С. 74–115. (Under the banner of Marxism. Philosophical and socio-political journal. No. 1. 1933. P. 74–115).

was carried out. Thus, the article reported that «at the disposal of the Marx-Engels-Lenin Institute there are about 900 pages of mathematical manuscripts of Marx in photocopies. All of them have now been deciphered and put in order. All the main works have also been translated into Russian. The manuscripts mainly consist of: 1) summaries of textbooks worked over by Marx (in which his own notes are not uncommon), 2) aggregated abstracts representing a summary (and often an overview) of all the material on one and the same question found by Marx in various sources, and finally 3) independent works, some of which exist only in initial drafts (often in several versions), while others are in drafts, rough copies, and finally in a completely fair-copy (from rough copies) form especially for Engels. There are few pages devoted to actual calculations – barely two dozen¹¹². This systematization of the manuscripts reflected the stages of scientific knowledge characteristic to K. Marx: from receptive activity to the presentation of his own concept (study of sources, generalization of the knowledge gained around key problems, drafts, finished presentation).

The authors of the publication not only did not hush up the previous stage of preparing the manuscripts for publication, but also dwelt in detail on the analysis of E.J. Gumbel's work on mathematical manuscripts, quoting his article at length¹¹³. But the

¹¹² Под знаменем марксизма. Философский и общественно-политический журнал. № 1. 1933. С. 75–76. (Under the banner of Marxism. Philosophical and socio-political journal. No. 1. 1933. P. 75–76).

¹¹³ The work carried out by E.J. Gumbel on the mathematical manuscripts of K. Marx was not hushed up later either. See: Литературное наследство К. Маркса и Ф. Энгельса. История публикации и изучения в СССР / Сост. Е. Кандель. М.: 1969. С. 163. (The Literary Heritage of K. Marx and F. Engels.

assessment expressed by S.A. Yanovskaya in her article was negative: «The mathematical manuscripts had been available in photocopies at the Marx-Engels Institute since 1925. By 1927, a significant portion of the material had been deciphered, and quite conscientiously at that. However, this cannot be said either about the choice of material for deciphering or, especially, about the order in which the material was presented. First of all, the most valuable work of Marx, placed above under the title «Historical Essay» and representing, in a certain sense, the key to all the other works, remained completely undeciphered. It was listed among «excerpts from various authors, since the notation methods vary» (see the report «On the Mathematical Manuscripts of K. Marx» in the «Chronicles of Marxism». III (1927). P. 58), although there is not a single excerpt there, and the notation methods vary due to the fact that different stages in the development of differential calculus are being discussed. Along with this, explicit notes (relating to divergent series and to Taylor's theorem) ended up in «independent works». The works on Taylor's and Maclaurin's theorems were in particular disarray, where often one page was assigned to one work, and its immediate continuation to another. All this was explained, in addition to the careless attitude to the matter on the part of the old leadership of the Institute¹¹⁴, by a complete lack of understanding of Marx's mathematical works «being put in order». As proof that this is so, it is enough to cite the following assessment of these works from

History of Publication and Study in the USSR / Comp. E. Kandel. Moscow. 1969. P. 163).

¹¹⁴ Gumbel's name was not mentioned, and all responsibility for the failed publication was placed on the previous leadership of the IME.

the above-mentioned report «On the Mathematical Manuscripts of K. Marx»: «Those manuscripts that do not contain calculations or excerpts can be considered independent works of Marx. Only these make sense to publish. They contain a free exposition of what Marx read, combined with numerous chronological dates, and philosophical reflections on what he read, and in some cases Marx poses problems in a special way, characteristic only of him»¹¹⁵.

S.A. Yanovskaya reported how the work was divided between the preparers of the mathematical manuscripts for publication, D.A. Raikov and A.N. Nakhimovskaya: «The translation of the first and third parts was done by Comrade Raikov, the second part by Comrade Nakhimovskaya. The translation was edited jointly by Raikov and Yanovskaya. The detailed inventory of the manuscripts used in this article was compiled by Comrade Nakhimovskaya, who also did the main work of establishing the order of Marx's manuscripts and comparing the notes with the textbooks used by Marx»¹¹⁶.

The names of the decipherers of the mathematical manuscripts were also named: R.S. Bogdan (Mateika)¹¹⁷ and Jacob

¹¹⁵ Под знаменем марксизма. Философский и общественно-политический журнал. № 1. 1933. С. 75–76 (примечание). (Under the Banner of Marxism. Philosophical and Socio-Political Journal. No. 1. 1933. P. 75–76 (note).

¹¹⁶ Под знаменем марксизма. Философский и общественно-политический журнал. № 1. 1933. С. 115 (Under the Banner of Marxism. Philosophical and Socio-Political Journal. No. 1. 1933. P. 115).

Под знаменем марксизма. Философский и общественно-политический журнал. № 1. 1933. С. 115. (Under the Banner of Marxism. Philosophical and Socio-Political Journal. No. 1. 1933. C. 115.

¹¹⁷ In the 1926 correspondence, Gumbel sent her greetings in letters and asked about her.

Wildhaber¹¹⁸ Considering S.A. Yanovskaya's high assessment of the work of the decipherers («a significant part of the material was deciphered and quite conscientiously»), it is necessary to say more about Jacob Wildhaber and Roza Semyonovna Bogdan. An employee of the IME archive, Roza Bogdány (Matejka) (1903–1951), took part in deciphering the manuscripts since 1925. Rózsa Bogdány was the first wife of the Hungarian writer János Matejka (1895–1940). They were among the 400 people who arrived in the USSR in 1921 as a result of the agreement between the RSFSR, the Ukrainian SSR and Hungary on the exchange of prisoners of war and interned civilians of July 28, 1921¹¹⁹; both worked at the IME for some time. A total of 13 emigrants from Hungary worked at the IME, who were attracted here by Ernst Czóbel, among them was the philosopher Georg (György) Lukács (1885–1971)¹²⁰. Studying the mathematical manuscripts of K. Marx could not help but influence the fact that after their processing R.S. Bogdan entered and graduated from the Faculty of Mechanics and Mathematics of Moscow State University in 1936.

¹¹⁸ Their work was not forgotten even after 30 years. See: Литературное наследство К. Маркса и Ф. Энгельса. История публикации и изучения в СССР / Сост. Е. Кандель. М.: Политиздат. 1969. С. 145. (The Literary Heritage of K. Marx and F. Engels. History of Publication and Study in the USSR / Comp. E. Kandel. Moscow. 1969. P. 145.).

¹¹⁹ Документы внешней политики СССР. Т. IV. 19 марта 1921 г. – 31 декабря 1921 г. М. 1960. С. 242–246; Собрание узаконений и распоряжений Правительства за 1922 г. М. 1950. С. 185–188. (Documents of the Foreign Policy of the USSR. Vol. IV. March 19, 1921 – December 31, 1921. Moscow. 1960. P. 242–246; Collection of Laws and Orders of the Government for 1922. Moscow, 1950. P. 185–188).

¹²⁰ Желицки Б.Й., Желицки Ч.Б. Венгерские эмиграционные волны и эмигранты (середина XIX – конец 50-х годов XX века). М. 2012. С. 265. (Zhelitzky B.Y., Zhelitzky C.B. Hungarian Emigration Waves and Emigrants (mid-19th – late 1950s). Moscow. 2012. P. 265.)

Jakob Wildhaber¹²¹ was born in 1887 in Switzerland (Sargans) and was a mathematics teacher by profession. In 1924 he came to the USSR with F. Platten to work in the Swiss cooperative «Solidarity» organized on the Volga, after which he taught mathematics in Saratov. Then he moved to Moscow and in 1931–1935 he worked at the IME, deciphering Marx's mathematical manuscripts, and then moved to Moscow State University¹²². In 1938, J. Wildhaber went missing.

As for further work on the publication of the remaining part of Marx's mathematical manuscripts, according to E. Kolman, «at the IME, after Adoratsky's death, the directors changed, and all of them took the same indecisive, wavering position of reinsurers on the issue

¹²¹ *Fayet J.-F., Huber P.* Die Russlandschweizer ohne Schutz? Die IKRK-Mission in Moskau als «verdecktes Konsulat» 1921-1938 // Studien und Quellen – Études et sources – Studi e fonti 28, 2002. S. 153–187. (IKRK Internationales Komitee vom Rothen Kreuz – Международный комитет Красного Креста).

¹²² «Mathematics teacher Jakob Wildhaber arrived in Novaya Lava [a village in the Ulyanovsk region] in 1924, but like other university-educated émigrés, he used his stay in the countryside to look for another job in the city. After only six months, he left Novaya Lava and began teaching mathematics at the technical institute in Marxstadt [now Marx] on the Volga. From 1931 to 1935, Wildhaber worked as a research fellow at the Marx-Engels Institute in Moscow. He deciphered Marx's manuscripts. The institute's staff, which was responsible for publishing the works of Marx and Engels, had recently undergone a «purge» of all old employees in the wake of the Menshevik trial, giving Wildhaber an opportunity to advance up the career ladder. At the same time, he was engaged in an activity that later may have cost him his life: «Development of my own discovery in the field of artillery calculations. Work completed and handed over to the USSR Defense Commissariat for practical use». In the spring of 1936, for unknown reasons, Wildhaber left (he was now a Soviet citizen) for Engels-city, where he began teaching at a secondary school. His mother received her last letter in January 1938. She raised the alarm by writing to the Red Cross. Two letters sent to Engels-city were returned marked «return, left». Wildhaber is considered missing in action» (*Fayet J.-F., Huber P.* Die Russlandschweizer ohne Schutz? Die IKRK-Mission in Moskau als «verdecktes Konsulat» 1921–1938 // Studien und Quellen – Études et sources – Studi e fonti 28, 2002, S. 164–165).

of publishing Marx's mathematical manuscripts»¹²³. E. Kolman himself, speaking at international forums on mathematics: at the Second London International Congress on the History of Science and Technology and at the Ninth International Congress on Mathematics in Zurich in 1931–1932, informed the scientific community about the mathematical legacy of K. Marx even before its publication¹²⁴.

**14. Work on the publication of K. Marx's book
«Mathematical Manuscripts». Moscow: «Nauka».
1968.**

¹²³ Кольман Э. Мы не должны были так жить! М. 2011. С. 229. (Kolman E. Die verirrte Generation. So hätten wir nicht leben sollen !).

¹²⁴ См.: Кедров Б.М., Розенфельд Б.А., Юшкевич А.П. Эрнест Кольман (К 75-летию со дня рождения) // Вопросы истории естествознания и техники. № 2. 1969. С. 72; РГАСПИ. Ф. 17. Оп. 3. Д. 893. Л. 10; РГАСПИ. Ф. 17. Оп. 163. Д. 823. Л. 6. Академия наук в решениях Политбюро ЦК РКП(б) – ВКП(б) 1922–1952. М.: РОССПЭН. 2000. С. 107, 123, 125. (Kedrov B.M., Rosenfeld B.A., Yushkevich A.P. Ernest Kolman (On the occasion of his 75th birthday) // Questions of the history of natural science and technology. No. 2. 1969. P. 72; RGASPI. F. 17. Inventory 3. Case 893. Sh. 10; RGASPI. F. 17. Inventory 163. Case 823. Sh. 6. Academy of Sciences in the decisions of the Politburo of the Central Committee of the RCP (b) – CPSU (b) 1922–1952. M. 2000. P. 107, 123, 125). In London, E. Kolman gave a talk on the topic «On the unpublished manuscripts of K. Marx relating to mathematics, natural sciences and technology and their history». См.: Комиссия по истории знаний 1921–1932. Из истории организации историко-научных исследований в Академии наук. Сборник документов / Сост. В.М. Орёл, Г.И. Смагина. М. 2003. С. 50) (See: Commission on the History of Knowledge 1921–1932. From the history of the organization of historical and scientific research in the Academy of Sciences. Collection of documents / Comp. V.M. Orel, G.I. Smagina. Moscow. 2003. P. 50). E. Kolman's report at the conference in Zurich was called «Karl Marx's Justification of Differential Calculus» (Über Marxens Begründung der Differentialrechnung) in the Congress Section «Philosophy and History» // Verhandlungen des International Mathematical Congresses (Zurich. 1932) / Hrsg. W. Saxer. Bd. 2: Sektions-Vorträge. S. 349–451. Kolman also reported that at the congress the famous Swiss mathematician R. Fueter (1880–1950) showed interest in Marx's mathematical manuscripts).

After 1945, in the post-war years, the magazine «Pod Znamenem Marxizma» and the collector «Marxism and Natural Science», with Marx's mathematical manuscripts published in them, unfortunately, became a bibliographic rarity and were practically inaccessible to the reader, not to mention the fact that they were withdrawn from wide circulation for ideological reasons¹²⁵. The absence of Marx's mathematical works in scientific circulation gave rise to speculation that these works were classified. Nevertheless, Marx's mathematical manuscripts continued to attract the attention of researchers. Soviet and Russian mathematician and historian of science Konstantin Alekseevich Rybnikov (1913–2004) presented a report on these manuscripts at a meeting of the Moscow Mathematical Society and chose them as the topic of his doctoral dissertation. K.A. Rybnikov left memoirs from which it follows that a political decision was required to resume work on publishing manuscripts in the USSR: «Due to objective reasons, research into Marx's mathematical manuscripts proceeded rather sluggishly until 1950, when, at a mathematical congress in Budapest¹²⁶, attention was again drawn to them, primarily by mathematicians from socialist countries. Among the members of the very representative Soviet delegation, as its secretary, was the future well-known historian of mathematics, Konstantin Alekseevich Rybnikov. It was to him that a

¹²⁵ The magazine «Pod Znamenem Marxizma» was characterized as «a magazine tainted by enemies of the people», and the collector «Marxism and Natural Science» «was found to be tainted by an article by the Morganist-Mendelian Meller» (RGASPI. F. 71. Inventory 4. Case 177. Sh. 67).

¹²⁶ On the sending of the Soviet delegation, including A.K. Rybnikov, to the conference in Budapest, see: Академия наук в решениях Политбюро ЦК РКП(б) – ВКП(б) 1922–1952. М. 2000. С. 475. (The Academy of Sciences in the decisions of the Politburo of the Central Committee of the RCP(b) – VKP(b) 1922–1952. Moscow. 2000. P. 475).

group of congress participants from people's democracies turned. The reason was simple.

By this time, Yanovskaya's article «On the Mathematical Manuscripts of K. Marx», published in 1933, had become well known in the world mathematical community, and here stood her student. However, in 1933, Rybnikov was only a student and was studying the history of the development of the calculus of variations. The other members of the Soviet delegation¹²⁷ were also not well informed about Marx's mathematical research. However, this meeting with foreign colleagues had unexpected consequences. In 1950, Rybnikov, PhD, held the position of instructor in the science department of the Central Committee of the CPSU. The personal archive of the Rybnikov family contains handwritten memoirs of Konstantin Alekseevich about this period of work. He wrote that upon his return, he reported on this conversation to Dmitry Trofimovich Shepilov¹²⁸, who supervised international scientific seminars and congresses. Previously, he was in charge of the propaganda department of the Central Committee, was at one time the editor-in-chief of the newspaper «Pravda», and in 1950 was appointed inspector of the Central Committee and was in fact the personal assistant of G.M. Malenkov¹²⁹. Shepilov immediately took Rybnikov to a member of the Central Committee of CPSU Pyotr Nikolaevich Pospelov¹³⁰, in the near future secretary of the Central Committee of CPSU, director of the IMEL, who had already been

¹²⁷ The composition of the Soviet delegation: Vinogradov I.M. (head of the delegation), Kolmogorov A.N., Khristianovich S.A., Aleksandrov P.S., Shtokalo I.Z., Markov A.A., Nikolsky S.M., Korobov N.M., Rybnikov A.K. (secretary).

¹²⁸ D.T. Shepilov (1905–1995) – statesman and party figure, economist.

¹²⁹ G.M. Malenkov (1901–1988) – statesman and party figure.

¹³⁰ P.N. Pospelov (1898–1979) – Soviet party figure.

informed about the interest of the international mathematical community in Marx's mathematical manuscripts.

Of course, he also knew that IMEL had an almost complete collection of Marx's mathematical works (in photocopies). He said to Rybnikov: «You are a mathematician, you are a communist, you need to study these manuscripts. I will issue necessary instructions about this. From this day on, you will have a new order of the day. You will work half a day in the Central Party Archives (CPA), and the rest of the day in your department»¹³¹.

Work on preparing K. Marx's mathematical manuscripts for publication was started at IMEL in October 1950, and from then on it was carried out directly in the IMEL archive.

On February 4, 1953, S.A. Yanovskaya and K.A. Rybnikov sent a report to the IMEL Directorate «On Marx's Mathematical Manuscripts», in which they dwelt in detail on the structure and significance of Marx's mathematical manuscripts, contrasting their point of view with the approach of E.J. Gumbel to the systematization and dating of Marx's mathematical works¹³². If in the 1930s Yanovskaya reproached Gumbel for the low assessment of the significance of Marx's mathematical manuscripts, the incorrect choice of manuscripts for deciphering, then in the report of 1953 the

¹³¹ Ласковая Т.А., Рыбников К.К., Чернобровина О.К. Математические работы К. Маркса: столетняя история поиска, расшифровки и анализа // Вопросы истории естествознания и техники. 2021. Т. 42. № 1 С. 71–88. (Laskovaya T.A., Rybnikov K.K., Chernobrovina O.K. Mathematical works of K. Marx: a hundred-year history of search, deciphering and analysis // Questions of the history of natural science and technology. 2021. Vol. 42. No. 1 P. 71–88).

¹³² «It is not surprising, – wrote the authors of the report on E.J. Gumbel's study of Marx's mathematical manuscripts, – that all this 'work' ended with a libel on Marx's manuscripts, published in the «Chronicles of Marxism», III, 1927, and containing an assessment of these manuscripts as Marx's student exercises» (RGASPI. F. 71. Inventory 4. Case 177. Sh. 66).

criticism of Gumbel's work was continued. Yanovskaya believed that after the German mathematician's work with the manuscripts, they were brought into a chaotic state, sheets of different works were mixed up, Gumbel arbitrarily dated individual manuscripts, despite even the presence of the author's dating¹³³.

This report and the whole issue of a new edition of Marx's mathematical manuscripts became the subject of discussion at a meeting of the IMEL Directorate on March 21, 1953. The following speakers spoke at this meeting: IMEL Director D.G. Obichkin (1899–1981), Academician A.N. Kolmogorov (1903–1987)¹³⁴, Corresponding Member of the USSR Academy of Sciences A.A. Maksimov (1891–1976), IMEL Senior Researcher and Philosopher V.K. Brushlinsky (1900–1992), IMEL Senior Researcher and Philosopher V.M. Posner (1877–1957), Head of the CPA K.P. Abrosenko, Candidate of Physical and Mathematical Sciences (since May 1953 Associate Professor at Moscow State University) K.A. Rybnikov, and others.

D.G. Obichkin stated from the very beginning of the meeting: «I believe that it is necessary to prepare and publish the manuscripts of Marx's mathematical works as soon as possible; to give the broadest explanations extracted from Marx's works, to open wide access to these works, to popularize them. When publishing Marx's manuscripts, it is necessary to include here also the circle of works that are connected with Marx's manuscripts»¹³⁵. The question of how to publish the manuscripts was discussed: in the «Archive of

¹³³ RGASPI. F. 71. Inventory 4. Case 177. Sh. 65–89, 98–107.

¹³⁴ A.N. Kolmogorov is a Soviet mathematician, one of the greatest mathematicians of the 20th century.

¹³⁵ RGASPI. F. 71. Inventory 4. Case 177. Sh. 44.

K. Marx and F. Engels» or as a separate book. The question of whether to publish the mathematical manuscripts separately or together with «Dialectics of Nature», which Engels had spoken about, was discussed.

The meeting did not ignore questions of a practical nature. It was clear that IMEL would not be able to publish such a work on its own without involving specialists from the USSR Academy of Sciences or other specialized organizations. In addition, there was no doubt that it was impossible for mathematicians to solve such a labour-intensive task as preparing manuscripts for publication without interrupting their main work (both S.A. Yanovskaya and K.A. Rybnikov taught at Moscow State University). K.A. Rybnikov himself later recalled how Soviet scientists coped with this task:

«At that time, the head of the Marx Documents Department [CPA IMEL] was Viktor Alekseevich Radus-Zenkovich, an old Bolshevik¹³⁶. At first, he met Konstantin Alekseevich with caution, but soon they established good business relations. Most of the employees were older ladies, but each of them knew her business well. One of them, Nina Ilyinichna (her last name, unfortunately, was not preserved in Rybnikov's memoirs)¹³⁷, knew several languages, and most importantly, she had the art of deciphering Marx's handwriting, which was always a difficult task for all researchers. Viktor Alekseevich knew how to organize the work of his small department. Persistent mathematical work, research by documentary

¹³⁶ Viktor Alekseevich Radus-Zenkovich ([1878] 1877–1967) – Soviet party and government figure, from 1940 he worked at the Marx-Engels-Lenin Institute – Marx-Engels-Lenin-Stalin under the Central Committee of the All-Union Communist Party (Bolsheviks) – CPSU.

¹³⁷ Nina Ilyinichna Nepomnyashchaya (1895–1976) – employee of the Central Political Administration of the IML under the Central Committee of the CPSU.

historians and handwriting experts yielded good results. All the texts of the archive were restored (and there were more than 100 pages of them). The difficult task of highlighting purely abstract knowledge, original research and serious analytical developments of what Marx had gleaned from the sources was also solved. In addition, all the documents in the archive were finally translated into Russian. In May 1953, Rybnikov returned to work at Lomonosov Moscow State University. Almost immediately, he told academicians Andrei Nikolaevich Kolmogorov and Ivan Georgievich Petrovsky¹³⁸ about Marx's mathematical research. In addition, he shared his own results of their analysis and presented a dissertation plan «On the Works of K. Marx in Mathematics». By the way, Kolmogorov was a participant in the International Mathematical Congress held in Hungary in 1950. Both Kolmogorov and Petrovsky reacted favorably to his report, and most importantly, they visited IMEL, where they got acquainted with Marx's archive, and Kolmogorov visited IMEL several times. On June 25, 1954, Rybnikov successfully defended his dissertation for the degree of doctor habilitatus in Physical and Mathematical Sciences on the topic «On the Works of K. Marx in Mathematics». (interestingly, it took place exactly (to the day) thirteen years after he defended his Ph.D. thesis in 1941). The opponents were well-known historians of mathematics A.O. Gelfond, A.P. Yushkevich and S.A. Yanovskaya. Kolmogorov sent a positive review. In connection with his participation in the defense, it is worth recalling one of its episodes. When one of the authoritative historians

¹³⁸ Ivan Georgievich Petrovsky (1901 – 1973) – an outstanding Soviet mathematician.

of mathematics spoke with a greater degree of skepticism about Marx's manuscripts, Andrei Nikolaevich turned to him with the question: «Have you seen them? But I have!» and laughed with his characteristic «Kolmogorov» laugh. It would seem that after the successful defense of the dissertation, the question of Marx's mathematical manuscripts was exhausted. However, much remained unclear. What mathematical apparatus did Marx have, what sources did he rely on and, finally, the main thing is what tasks did he set for himself? In 1965, Rybnikov was sent on a business trip to London. The program of the trip included work in the library collections of the British Museum, which contained almost exhaustive information about the sources used by Marx, as well as in London and Cambridge universities. With the help of the Soviet embassy in London, it was found out that traces of Marx's library searches should be sought in the colleges of Trinity, St. John's and St. Catharines. Incidentally, unknown pages of Marx's works also appeared. For example, two years before the trip, the head of the Marx and Engels Documents Section, Olga Konstantinovna Senekina, managed to obtain a number of photocopies of unknown manuscripts at the International Institute of Social History in Amsterdam. As for the London expedition, the legendary Soviet diplomat, former USSR Ambassador to Great Britain Ivan Mikhailovich Maisky ¹³⁹ provided invaluable and qualified assistance in its organization. His extensive network of diplomatic and academic acquaintances in London enabled him to

¹³⁹ I.M. Maisky (1984–1975) – Soviet diplomat, historian and publicist.

provide advice on the organisation of work in library collections, primarily those of the British Museum»¹⁴⁰.

Among the documents of the RGASPI there are also other documentary evidence of the resumption of work by Soviet mathematicians on the publication of a book with Marx's mathematical manuscripts. On March 31, 1953, the director of IMEL G.D. Obichkin reported to the secretary of the Central Committee of the CPSU P.N. Pospelov: «During 1950-1953, Professor S.A. Yanovskaya of the Lomonosov Moscow State University and Associate Professor K.A. Rybnikov systematized and studied the mathematical manuscripts of K. Marx. K. Marx's work on mathematics is of great scientific importance for the natural sciences in general and for mathematics in particular.

The IMEL Directorate at a meeting on March 21 of this year [1953] discussed the issue of K. Marx's mathematical legacy and decided to prepare K. Marx's mathematical manuscripts for publication as a separate edition.

The IMEL Directorate requests your instructions on seconding a PhD, associate professor comrade Rybnikov K.A. to work at IMEL on preparing the mathematical manuscripts of K. Marx for publication.

Comrade Rybnikov has been a member of the CPSU since 1944 and... Comrade Rybnikov has agreed to work for IMEL»¹⁴¹.

¹⁴⁰ Ласковая Т.А., Рыбников К.К., Чернобровина О.К. Математические работы К. Маркса: столетняя история поиска, расшифровки и анализа // Вопросы истории естествознания и техники. 2021. Т. 42. № 1 С. 71–88 . (Laskovaya T.A., Rybnikov K.K., Chernobrovina O.K. Mathematical works of K. Marx: a hundred-year history of search, deciphering and analysis // Questions of the history of natural science and technology. 2021. Vol. 42. No. 1 Pp. 71–88).

¹⁴¹ RGASPI. F. 71. Inventory 5. Case 345. Sh. 15.

Olga Konstantinovna Senekina, an employee of the CPA IML, and mathematician Anatoly Zalmanovich Ryvkin (1912–1979), editor of the «Nauka» publishing house, also took part in the work on the book *Mathematical Manuscripts of K. Marx*. Academics A.N. Kolmogorov and I.G. Petrovsky shared their comments and advice.

Unfortunately, S.A. Yanovskaya did not live two years to see the publication of K. Marx's «*Mathematical Manuscripts*». They were published in 1968. The edition included both K. Marx's manuscripts on mathematics themselves and their interpretation, as well as an explanation of those places where Marx's reasoning seemed insufficient to mathematicians. Anticipating the publication of this edition, mathematician and economist, son of one of the preparators Albert Anatolyevich Ryvkin (1940-2003) described in the magazine «*Priroda*» (Nature) as follows the activities of K. Marx in mathematics: «Marx concentrated his efforts in the area that is today called the foundations of mathematics». He named three main difficulties in preparing manuscripts for publication: the difficulty of reading Marx's handwriting, abbreviations, notes in the margins, searching for sources that K. Marx used, linking Marx's polemical notes to their real basis, and finally, it was necessary «to correctly understand the thoughts expressed by Marx, to find out the reasons that forced him to turn to this or that problem, to systematize the pages of manuscripts written at various times, to outline the scientific and historical atmosphere in which Marx worked»¹⁴². In the article

¹⁴² *Рывкин А.А.* Математические рукописи Маркса (К 150-летию со дня рождения Карла Маркса) // *Природа*. № 5. 1968. С. 16. (Ryvkin A.A.

by A.A. Ryvkin, the work of S.A. Yanovskaya on the preparation of the publication was highly appreciated. It was she who provided each manuscript with comments and wrote a detailed introductory article.

The texts in the publication were given both in German and in parallel in Russian translation. The structure of the book with mathematical manuscripts was different from the one proposed by E.J. Gumbel for his partial publication. Thus, the 1968 edition was divided into two sections. The first part, «Differential Calculus, Its Nature and History», included two manuscripts by Marx on differential calculus: «On the Concept of a Derived Function» and «On the Differential», sketches for the latter manuscript, as well as the manuscript «On the History of Differential Calculus». Then followed fragments from manuscripts devoted to the exposition of the theorems of Taylor and MacLaurin, the theory of analytic functions of J.L. Lagrange, and the analysis of the method of J.L. D'Alembert. The second part, «Description of Mathematical Manuscripts», was in turn divided into parts and contained fragments of Marx's manuscripts on mathematics according to the periods of their writing: before 1870, the 1870s, and the 1880s, with extensive commentary by Soviet mathematicians on these fragments. The publishers supplied the work with an appendix in which they set out in several articles how the concepts that were the focus of Marx's attention had been treated in the history of mathematics. The preface did not mention E.J. Gumbel's work on the manuscripts, which had begun in 1925, since it had already been the subject of consideration in 1933.

15. Further work on the publication of the mathematical manuscripts of K. Marx

At present, the 1968 publication is the most complete and authoritative edition of the mathematical manuscripts of K. Marx. It is on its basis that the mathematical manuscripts of K. Marx were published in different countries in different languages and even in German in Germany¹⁴³. In this case, mainly the first part of this work, «Differential Calculus, Its Nature and History», was published. (See: K. Marx, *Mathematical manuscripts*. Tokyo 1949. (in Japanese), Marx, *Mathematical manuscripts*. Tokyo 1973. (in Japanese), K. Marx, *Mathematical manuscripts*. A cura di F. Matarrese e A. Ponzio. Bari 1975, K. Marx, *Matem. rukopisy*. Prague 1978, K. Marx, *Mathematical manuscripts*. London 1983, K. Marx, *Les manuscrits mathematiques de Marx. Etude et pres. Par A. Alcouffe. Premier trad. francaise*. Paris. 1985 and others.

The exception is the publication K. Marx. *Mathematical manuscripts*. Calcutta: Viswakos Parisad. 1994. Here are published not only Marx's independent manuscripts on mathematics, but also a description of his notes; and letters from K. Marx and F. Engels are also provided, which serve as a source for studying the history of Marx's study of this science. However, the basis for this type of publication was again the publication of *Mathematical Manuscripts* in 1968.

¹⁴³ *Endeman W.* Marx Karl. *Mathematische Manuskripte*. Kronbergs Scriptor. 1974.

Edition of the academic Complete Works of K. Marx and F. Engels in the original languages (MEGA²), the decision on which was made in 1966 in the Central Committee of the CPSU and the Central Committee of the SED, assumed the publication of Marx's mathematical studies in full. His independent works on mathematics were to form the content of volume 28 of the first section of MEGA², combining philosophical, historical and political works, articles and sketches that came from the pen of K. Marx and F. Engels. Marx's works on mathematics of an excerptive nature were supposed to be included in volume 30 of the fourth section of MEGA² with extracts, notes and notes on books¹⁴⁴. The priority of Soviet scientists in the previous work on the publication of K. Marx's mathematical manuscripts also determined that within the framework of cooperation in the publication of MEGA² volumes of specialists from the USSR and scientists from the GDR, K. Marx's mathematical manuscripts should be prepared for publication by Soviet mathematicians. In the mid-1980s, collaboration on this issue was offered to a group of mathematicians from Moscow State University under the leadership of A.K. Rybnikov, who, as has already been said earlier, worked together with S.A. Yanovskaya on the publication of mathematical manuscripts of 1968. A.K. Rybnikov himself, for obvious reasons, refused to undertake the new painstaking preparation of manuscripts for publication. But the work

¹⁴⁴ Some of Marx's mathematical calculations have already been published in MEGA. See: *Marx K. Formeln und Berechnungen zu Mehrwert- und Profitrate // MEGA². Bd. II/14. S. 3–7; Derselbe. Rate des Mehrwerts und Profitrate // Ebenda. S. 8–18; Derselbe. Mehrwertrate und Profitrate mathematisch behandelt // Ebenda. S. 19–150.*

of other mathematicians¹⁴⁵ did not take place either. Thus, unfortunately, it was not possible to create a team of mathematicians and textual critics whose collective efforts could have been used to prepare K. Marx's manuscripts for MEGA taking into account not simple MEGA's editorial principles, which included the reproduction of all variants present in the manuscripts and corrections made during the scientific editing of K. Marx's texts, and then to comment on them in detailed notes¹⁴⁶. Nevertheless, by the mid-1990s, the Sector of Works of K. Marx and F. Engels of the IML textual work on the manuscripts selected for publication in volume I/28 was completed. The decipherment of individual passages in the manuscripts was clarified by G.B. Kovgankin (1925–1992)¹⁴⁷, an employee of the CPA IML. But the work stopped there. It must be acknowledged that political events in the USSR in the 1980s and 1990s had no impact on the pace and results of work on the preparation of the MEGA volumes.

It did not progress after the mid-1990s, when the French scholar Alain Alcouffe took on the task of completing the work on publishing the mathematical manuscripts, having prepared for publication the edition of K. Marx, *Les manuscrits mathématiques de Marx. Etude et présentation*. Par A. Alcouffe. Premier trad. française. Paris. 1985.

¹⁴⁵ MSU–mathematicians A.S. Kuzichev and Z.A. Kuzicheva were also invited to participate in the work.

¹⁴⁶ It is necessary to emphasize once again that in the preparation of mathematical manuscripts within the framework of MEGA, a team of preparators was never created and all those who took part in this work did not work together, but at different times.

¹⁴⁷ *Bagaturija G.*: Geliy Borisovič Kovgankin. Nachruf // MEGA-Studien. Hrsg. von der Internationalen Marx-Engels-Stiftung. Amsterdam. 1994/1. Berlin: Dietz Verlag, 1994. S. 182–183.

When a new (already fourth) attempt was made to publish the manuscripts within the framework of MEGA², which was started by C.-E. Vollgraff, the publishers doubted the legitimacy of selecting certain mathematical works by K. Marx for publication in volume I/28. In other words, the question arose again, the answer to which researchers had been trying to find for almost a century: for what purpose did Marx undertake his research in mathematics and what in mathematical manuscripts should be considered excerpts, and what his independent contribution to this science. At this point, work on this volume of MEGA² stopped. And to this day this question remains open.